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IN MEASURE, NUMBER, AND WEIGHT

STUDIES IN MATHEMATICS AND CULTURE

JENS HØYRUP

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PREFACE

"Mathematics and culture" is a phrase that may be interpreted in numerous ways. A bit more precision is achieved when the concept of "culture" that is involved is specified to be the one which is current among anthropologists.

In order to characterize my approach I have therefore often spoken of the "anthropology of mathematics." Even this phrase does not correspond to any discipline or generally known field of interest, nor will it probably ever do so. Then what do I mean by it?

Twelve years ago, when using the term for the first time, I explained (H0yrup 1980: 9) that I did so

> because of dissatisfaction with the alternatives. History and social history of mathematics both tend as ideal types to concentrate on the historically particular, and to take one or the other view (or an eclectic combination) in the internal-external debate when questions of historical causality turn up. "Historical sociology" would point to the same neglect of cognitive substance as present in the sociology of science. "Sociology of mathematical knowledge" would suggest both neglect of the historically particular and a relativistic approach to the nature of mathematical knowledge, which may be stimulating as a provocation but which I find simplistic and erroneous as it stands.1
>
> What I looked for was a term which suggested neither crushing of the socially and historically particular nor the oblivion of the search for possible more general structures: a term which neither implied that the history of mathematics was nothing but the gradual but unilinear discovery of ever-existing Platonic truths nor (which should perhaps be more emphasized in view of prevailing tendencies) a random walk between an infinity of possible systems of belief. A term, finally, which involved the importance of cross-cultural comparison.
>
> The latter term suggested social anthropology, a discipline whose cognitive structure also seemed to fulfil the other requirements mentioned [. . .] .

This approach is one which makes me feel like a sociologist among historians, a cross-breed between a philosopher and a historian among sociologists, and a bastard historian among philosophers. It makes the actual content of mathematics, in particular the changing mode of mathematical thought, stand out as crucially important, both for the functions mathematics can fulfill, and for the way the pursuit and development of mathematics is conditioned by the wider social and cultural context. Reduced to essentials, (my brand of) the anthropology of mathematics is thus an approach to the history of mathematics that, first, rejects the distinction between "internalism" and "externalism"; second, even when investigating the contributions of individuals, sees these as members of a particular culture, or rather as members of one or perhaps several intersecting subgroups within a general cultural matrix; third, tends to use the evidence which can be found in the production of individuals as anthropologists use the testimonies furnished by their informants.

The question of externalism versus internalism earns further discussion. This, however, presupposes some preliminary considerations on the concept of causation.

General quasi-philosophical lore distinguishes two concepts: The "Humean" cause and the "Aristotelian" cause, of which the Humean cause is said to correspond to the "efficient" Aristotelian cause and to make the remaining Aristotelian causes superfluous.

This is wrong already for the reason that Hume *(Enquiries Concerning Human Understanding* VII,ii:59) considers causation to be simply *an expectation on the part of the observer* produced by habit. Leaving this finesse aside, and identifying Hume with the "Hume" of quasi-philosophical folklore, being hit by billiard ball *A* is the Humean reason that ball ß starts to roll.

To this an Aristotelian will object that there are many answers to the question why *B* moves as it does. Being hit is evidently one; but if the balls had consisted of soft clay the outcome would have been different; so it would if A and *B* had not been spherical, or if *B* had been located at the very edge of a table not provided with a cushion. A complete answer to the question *why* will thus involve efficient causes (the hitting); material causes (ivory, not clay; the surface of the cloth); and formal causes (the laws of semielastic impact and of sliding/rolling, as well as the geometrical forms involved). It will also have to mention that somebody plays billiards and wants *B* to move (perhaps as it does, perhaps otherwise), ultimately wanting to win the game and to gain the stake; both of these are final causes.

Thus causations are manifold, as Aristotle remarked, even though they can be grouped in classes according to their character (*Physica* 195[a]28). Only the folklore and the Aristotelian textbook tradition speak in the singular about *the* efficient, *the* material, *the* formal, and *the* final cause.

Evidently, the rigid framework of precisely four causes is of scant value if we want to explain historical processes. Even more irrelevant, at least when we discuss the history of culture, ideas, or the sciences, is the single "Humean" cause. It is the search for such single, instantaneous, and thus efficient causes which produces stories like Newton's falling apple. Even if the anecdote had been true and the observation in the garden had indeed been the efficient cause of—i.e., the occasion for—Newton's formulation of the law of gravitation, this answer would only be of interest for the psychology of scientific creativity (and then only within the framework of Newton's total mental make-up); different questions are asked in the history of ideas and of the sciences—questions that have much more to do with the remaining Aristotelian headings.

This brings us back to the problem of internalism and externalism. From time to time we are told that internalist explanations are real explanations because they alone are ("Humean") causal; the socio-cultural context in which scientific development takes place is of course as necessary as the soil is necessary for the forest (cf. Whitehead 1926: 23), yet context and soil are "incidents," not *causes.* This justification, however, is untenable; the results of earlier science and the questions raised by these results are just as much background to the actual events in scientists' lives as the sociocultural context. The (still irrelevant) efficient causes of their doings are just as likely to be found outside the framework of scientific ideas and results (say, in their job situation) as inside (the reading of a particular book or a discussion with a colleague at a particular moment).

A better justification is provided by a "local separation of variables." Looking at what goes on during a particular epoch one may quite legitimately take the actual institutional and cultural framework within which scientists move for granted and as relatively constant, and look at how scientists react to and continue the scientific tradition that they encounter—or, just as legitimately, one may take the level and character of the science of the time as a given and look at how it is able to respond to social needs and how it is stimulated or hampered by sociocultural circumstances, pursuing thus the externalist road.

Valid as this justification is, it also shows the restricted validity of both the internalist and the externalist approach as only first approximations. As soon as the development over longer periods or the comparison between different cultures or epochs are undertaken, the character and substance of scientific thinking and the aims pursued by the sciences, as well as institutions, ideologies, and general social needs, change. This will not prevent the historian from making (valid) internally or externally oriented *descriptions* of events, or some eclectic mixture—but it deprives such descriptions of explanatory capability, as long as no dialectical synthesis takes place.

The practical necessities of exposition prevent most of us from honoring such ideal claims in full. In the following essays, accordingly, the content side of mathematics is mostly dealt with in rather general terms (concentrating on cognitive organization and mode of thought), and emphasis is on its interaction with the sociocultural setting. Studies where I investigate *the mathematics* of various cultures have been omitted from the present collection and only appear in the footnotes. The collection as it presents itself thus verges more toward externalism than it should ideally do.

The collection contains in total eight essays, the individual publishing histories of which are told in the corresponding introductions. Each essay starts from a definite perspective or a set of specific, acknowledged questions. It is, indeed, impossible to ask about everything at one time, and the claim that one makes (for example) history *simpliciter* is at best naive. However, the attempt to answer questions asked from one perspective will by necessity raise other questions and thereby introduce new perspectives: thus also here. The perspectives of essays written at a later date may therefore be complements, at times perhaps correctives, to others written before. In this way, I hope, a more comprehensive picture of the "anthropology" of mathematics will emerge from the totality than the one that presents itself in its single constituent parts.

The ordering of the essays is thematic, and does not correspond to the order in which they were written, nor to the dates of publication. A first cluster consists of essays that are primarily sociological or anthropological in orientation (Chapters 1-4). Chapter 1 compares features of Sumero-Babylonian, ancient Greek, and Latin medieval mathematics, whereas Chapter 2 is an attempt to trace the specific character and the history of pre-Modern practitioners' mathematics, which I characterize by the term *subscientific.* Chapter 3 investigates the interplay between state formation processes, scribal culture, and mathematics in ancient Mesopotamia, and Chapter 4 explores the specific character of Islamic mathematics and the sociocultural roots of its particular accomplishments: an unprecedented synthesis between mathematical theory and practice. Together, the four essays may be read collectively as an attempt to delineate some of the main facets of early "Western" mathematics.[2]

Chapters 5-7 probe the impact of ideologies, ideas, and philosophy on mathematics from the Latin High Middle Ages through the late Renaissance. Chapter 5 does so broadly, asking in particular to what extent formal philosophies and quasi-philosophical attitudes influenced the aims and ideals pursued by "mathematicians" from the early twelfth through the late sixteenth century. Chapter 6 studies the thirteenth-century mathematical author Jordanus de Nemore through his works (nothing is known directly about the person), tracing how he oriented himself with regard to the contradictory currents and attitudes that surrounded him. Chapter 7 scrutinizes the received persuasion that Platonism was a decisive motive force in the development of

Renaissance mathematics, suggesting the alternative thesis that the dominant ideology of humanist mathematics can be characterized as "Archimedism" and trying to trace the changing form and the impact of this ideology.

All these essays deal with the anthropology of pre-Modern mathematics. Chapter 8, written in collaboration with my friend and colleague Bernhelm Booß, focuses on the character and setting of Modern and contemporary mathematics. It does so through the perspective defined by the impact of militarization and warfare on mathematics. It may justly be argued that this perspective is not only limited but also narrowing and even distorting. We have exerted ourselves, however, not to use the distorting mirror for the purpose of caricature but in order to make visible and understandable features of contemporary mathematics that tend to be neglected; and, also important, to see which features are not deformed even by this disfiguring strain, and why.

Across these divisions, four recurrent themes (beyond the rejection of the internalist/externalist dichotomy) run through the book. I shall list them without arguing here for their pertinence—it is sufficiently done in what follows (so I hope), inasmuch as at all necessary.

One theme can be characterized as *desacralization without denigration.* The alternative to seeing mathematics as an ever-existing Platonic truth toward which mathematicians of all epochs strive when not hampered by obstructive forces need not be total relativism.

Another theme is that *actors participate in institutions,* not only being shaped by these but also shaping them. Thus, institutions mediate the influence of general sociocultural forces on actors—but actors, to the extent that institutions are not totally rigid, also contribute to the shaping of these in interaction with the general sociocultural forces to which they are also submitted through other channels.

The third theme is the dialectic between tradition and actual situation. In mathematics, as in other branches of culture, what is done in one generation presupposes what has been done before; but it presupposes it *through the form in which it is actually known and understood.* The tradition is always understood—which by necessity means misunderstood—through concepts and mental habits formed through an actual practice; but coming to grips with the tradition is in itself an essential part of the practice of (*in casu*) the mathematician, thus contributing to the formation of concepts and habits.

The fourth theme explains the title of the book: *The relation between "high" and "low" knowledge. Measuring, counting* and *weighing* are indeed the (most practical and hence "low") starting points for mathematics. The phrase *in measure, number, and weight,* however, is borrowed from Wisd. 11:21, where it describes the principle of the Lord's Creation (quoted by almost every Christian author between Augustine and Pascal writing about the importance of mathematics). This transformation of "low" into "high" knowledge is a constant characteristic of pre-Modern mathematics, whereas the reverse process, after modest beginnings in ancient Alexandria and rise to equal prominence during the Islamic Middle Ages, became the cardinal ideology of utilitarian mathematicians from the late sixteenth century onward while remaining in actual reality only one facet of a twin movement.

With one exception, the single essays carry dedications, which are those of the original publications. Some of them are of private but most of public character. Insofar as I have considered them to be of public interest yet not self-explanatory, the dedications are explained in the introductions of the individual parts.

The book as a whole I dedicate to my mother, and to the memory of my father. This I could do for many reasons—but the one I will mention on the actual occasion is that rich stimulation of my intellectual curiosity which I received from them.

I also dedicate it to the memory of my beloved wife Ludovica, who was so eager to have this book published, and without whose enthusiasm and tender support I might never have completed it.

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1. "Varieties of Mathematical Discourse in Pre-Modern Sociocultural Contexts: Mesopotamia, Greece, and the Latin Middle Ages." *Science & Society* 49 (1985), 4-41. © 1985 *Science & Society*.
2. "Sub-Scientific Mathematics. Observations on a Pre-Modern Phenomenon." *History of Science* 28 (1990), 63-86. © 1990 Science History Publications.
3. "Mathematics and Early State Formation, or, The Janus Face of Early Mesopotamian Mathematics: Bureaucratic Tool and Expression of Scribal Professional Autonomy." Revised contribution to the symposium "Mathematics and the State," 18th International Congress of History of Science, Hamburg/Munich, 1-9 August 1989. *Filosofi og videnskabsteori på Roskilde Universitetscenter.* 3. Række: *Preprints og Reprints* 1991 nr. 2.
4. "The Formation of "Islamic Mathematics": Sources and Conditions." *Science in Context* 1 (1987), 281-329. © 1987 Cambridge University Press.
5. "Philosophy: Accident, Epiphenomenon, or Contributory Cause of the Changing Trends of Mathematics. A Sketch of the Development from the Twelfth Through the Sixteenth Century." *Filosofi og Videnskabsteori pä Roskilde Universitetscenter.* 1. Raekke: *Enkeltpublikationer* 1987 Nr. 1. A Croatian translation has appeared as "Filozofija: Slucaj, epifenomen ili sinergijski uzrok promjene trendova u matematici. Obriz razvitka od dvanaestoga do sesnaestoga stoljeca." *Godisnjam zapovijest filozofije 5* (Zabreb 1987), 210-74.
6. "Jordanus de Nemore: A Case Study on 13th-Century Mathematical Innovation and Failure in Cultural Context." *Philosophica* 42 (Ghent, 1988), 43-77. © 1988 *Philosophica.*
7. A somewhat different version has appeared as "Archimedism, not Platonism: On a Malleable Ideology of Renaissance Mathematicians (1400 to 1600), and on Its Role in the Formation of Seventeenth-Century Philosophies of Science," in C. Dollo (ed.), *Archimede: Mito Tradizione Scienza.* (Biblioteca di Nuncius. Studi e testi, IV). Firenze: Olschki, 1992. The present text was prepared for the symposium "Renaissance-

Philosophie und neuzeitliches Denken III: Die klassischen Interpretationen im Lichte der modernen Forschung," Dubrovnik, 1990.

8. An earlier version appeared in German as: Bernhelm Booß & Jens H0yrup, *Von Mathematik und Krieg. Über die Bedeutung von Rüstung und militärischen Anforderungen für die Entwicklung der Mathematik in Geschichte und Gegenwart.* (Schriftenreihe Wissenschaft und Frieden, Nr. 1). Marburg: Bund demokratischer Wissenschaftler, 1984. A draft of the present text was prepared by Jens H0yrup for the Sixth International Congress on Mathematical Education, Fifth Special Day on Mathematics, Education, and Society, Budapest, July 31, 1988.

It is a pleasure to express my gratitude for the permission to republish the papers in the present context.

The form in which the essays appear herein is in *grosso modo* unchanged (with the reservation concerning 7 and 8, which was already stated). The reference system, however, has been harmonized and unified, and in part updated (in particular concerning autoreferences to preprint versions that have been published in the meantime), and cross-references between the essays have been inserted. Some linguistic filtration has also taken place. The subject matter, on the other hand, has not been tinkered with.

References are mostly made according to the author-date system. A few standard abbreviations are used, and articles from encyclopedias (in particular *DSB—Dictionary of Scientific Biography)* are referred to by author and title; further identification is given in the bibliography. Many primary sources are also referred to by author (if known) and title; in such cases, the bibliography contains a cross-reference to the edition I have used. Other primary sources are directly referred to by editor/translator-date.

All translations from original languages where nothing else is indicated are mine. An indication such as "Russian trans. Krasnova 1966" means that I have translated from the Russian translation in question, whereas "trans. Colebrooke 1817" indicates that I quote Colebrooke's English translation.