MATHEMATICS AND EARLY STATE FORMATION
or
THE JANUS FACE OF EARLY MESOPOTAMIAN MATHEMATICS: BUREAUCRATIC TOOL AND EXPRESSION OF SCRIBAL PROFESSIONAL AUTONOMY

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Kenneth O. May in memoriam

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TIMETABLE

3500	
	Uruk V
3300	
	Uruk IV
3100	
	Uruk III/Djemdet Nasr
2900	
	Early Dynastic I
2750	
	Gilgameš
	Early Dynastic II
2600	
	Fara
	Early Dynastic III
2350	
	Akkadian period
2200	
	Gudea
2100	
	Ur III
2000	
	Old Babylonian period
1750	Hammurapi
1600	

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Preface

The following essay is the result of an invitation to present »something Babylonian« at the symposium »Mathematics and the State« at the XVIIIth International Congress of History of Science, Hamburg / Munich, 1st-9th August 1989. I took advantage of the opportunity to attempt a synthesis of a number of approaches to the »anthropology« of Mesopotamian mathematics, each concentrating on specific aspects, in which I have engaged myself at various occasions during the last decade. Evidently I have made no effort to repeat everything which I have said at these earlier occasions on the subject; on the contrary, the attempt at synthesis has led me to change quite a few formulations and to shift the emphasis at certain points. Furthermore, of course, new epigraphic and archaeological material as well as new interpretations of familiar sources have come up during the 1980es. I will certainly not be aware of everything, especially not outside the domain of mathematical texts; none the less, what has come to my knowledge since 1980 weighs heavily at several points.

Of special importance has been the series of Berlin Workshops on Concept Development in Babylonian Mathematics (four to date). As it will be clear from the references, the synthesis draws extensively on work done by the members of this workshop, in particular on the works of Peter Damerow, Robert Englund, Jöran Friberg, Hans Nissen and Marvin Powell. It is a pleasure for me to express my gratitude to all of them for inspiration, discussions and invaluable information. I am also thankful to Denise Schmandt-Besserat for her constant efforts to keep me oriented on her results by means of offprints; to Michael Boakye-Yeadom, Pernille Jensen, Charlotte Justesen, Lucca Weis Kalckar, Morten Hjort Mikkelsen and Carsten Smith Petersen, who gave me the occasion to supervize a student project on state formation theory and state formation in early Mesopotamia in the Spring term 1989; and (as so often!) to the staff of the interlibrary

service of Roskilde University Library, without whose kind and effective assistance I would never have been able to engage in Mesopotamian studies.

Special thanks are due to Herbert Mehrtens and Walter Purkert, organizers of the symposium »Mathematics and the State«. Had it not been for their invitation to the symposium I would certainly not have undertaken anything as venturesome as a global analysis of the relation between Mesopotamian mathematics and the social and cultural forces moulding and moulded by the early Mesopotamian state.

I dedicate the work to the memory of Kenneth O. May, who in 1974 commented upon my first amateurish attempt at broad historical syntheses that although he agreed with my general thesis and found the generalizations plausible, what was needed was specific examples in which the interactions between mathematics and other phases of culture was »*traced out* and *verified in detail*« (his emphasis). I hope the present work would have been to his taste.

I. Mathematics and the early state

In his famous and somewhat notorious book on »Oriental Despotism«, Karl Wittfogel [1957:29f] presented a simple thesis connecting the first development of mathematics and astronomy with the rise of the early »Oriental« state —*viz* that the state was »hydraulic«, i.e., developed in order to plan large-scale irrigation, and that mathematics and mathematical astronomy were created for that purpose:

(A) The need for reallocating the periodically flooded fields and determining the dimension and bulk of hydraulic and other structures provide continual stimulation for developments in geometry and arithmetic. [...] Obviously the pioneers and masters of hydraulic society were singularly well equipped to lay the foundations for two mayor and interrelated sciences: astronomy and mathematics.

> As a rule, the operations of time keeping and scientific measuring and counting were performed by official dignitaries or by priestly (or secular) specialists attached to the hydraulic regime. Wrapped in a cloak of magic and astrology and hedged with profound secrecy, these mathematical and astronomical operations became the means both for improving hydraulic production and bulwarking the superior power of hydraulic leaders.

This thesis is in fact widely held, though often in less outspoken and rigid form. As also observed by Wittfogel, it was already proposed by Herodotos to explain the presumed Egyptian origin of geometry. My reason to take Wittfogel's very explicit statement as my starting point is that it exposes the problematic nature of the conventional thesis so clearly. If we concentrate on Mesopotamia, Wittfogel is wrong on all factual accounts (Egypt would come out no better):

— Irrigation systems only became a bureaucratic concern (and then only in certain periods) many centuries after the rise of statal bureaucracy (which took place in the later fourth millennium[1]). No doubt the

[1] The date is B.C., of course, like all dates in the following. And approximate, like

irrigation economy provided the surplus needed to feed the bureaucracy; but it was taken care of locally, and often by kin-based communities (as it often is even in today's Iraq)[2].

— Old Babylonian mathematical texts (c. 1700 B.C.) deal with construction of irrigation works, but only with the need for manpower, the wages to be paid, and the volume of earth involved. The dimensions of the constructions were not determined mathematically.
— Neither the sacred nor the secular calender were ever involved in irrigation planning in Mesopotamia.
— Mathematical astronomy was only created almost 3000 years after the rise of the state, and was concerned with the moon and the planets, i.e., irrelevant for irrigation planning.
— Even astrology is a late invention. Only in the first millennium are bureaucratic computation and occult endeavours of any sort connected through a common group of practitioners.

The easy version of the connection between the rise of the state and the development of mathematics (in Mesopotamia and elsewhere) is thus an illusion. In order to approach the issue in a profitable way we will have to ask some apparently trite questions: what *is* a state, and what *is* mathematics—if we are to discuss the two entities in the perspective of the Bronze rather than the Atomic Age.

II. The early state, and its origin

In his book, Wittfogel points [*ibid.*, 383-386] to two classical approaches to the problem of early state formation—both due to Friedrich Engels. Engels summarizes the thesis of *Die Ursprung der Familie, des Privateigentums und des Staats* as follows (MEW XXI, 166f):

all dates below!

[2] See, e.g., [R. McC. Adams 1982], and [C. C. Lamberg-Karlovsky 1976:62f].

(B) Da der Staat entstanden ist aus dem Bedürfnis, Klassengegensätze im Zaum zu halten, da er aber gleichzeitig mitten im Konflikt dieser Klassen entstanden ist, so ist er in der Regel Staat der mächtigsten, ökonomisch herschenden Klasse, die vermittelst seiner auch politisch herrschenden Klasse wird und so neue Mittel erwirbt zur Niederhaltung und Ausbeutung der unterdrückten Klasse.

In *Anti-Dühring,* on the other hand, he considers the state as »Verselbständigung der gesellschaftlichen Funktion gegenüber der Gesellschaft« which then, as the opportunity presented itself, changed from servant to master, be it »als orientalischer Despot oder Satrap, als griechischer Stammesfürst, als keltischer Clanchef u.s.w.«, but where it shall still be remembered that »der politischen Herrschaft überall eine gesellschaftsliche Amtstätigkeit zugrunde lag« (MEW XX, 166f).

Both points of view are present in the standard references of modern political anthropology. According to Morton Fried's *Evolution of Political Society,* the state arises as »a collection of specialized institutions and agencies, some formal and others informal, that maintain an order of stratification« [Fried 1967:235], where a »stratified society« itself is understood as one »in which members of the same sex and equivalent age status do not have equal access to the basic resources that sustain life« [*ibid.,* 186]—i.e., in a generalized sense, a class society. Elman Service, on the other hand, sees statal organization as the result of a quantitative and often gradual development from »relatively simple hierarchical-bureaucratic chiefdoms, under some unusual conditions, into much larger, more complex bureaucratic empires« [1975:306]. The chiefdom itself is a hierarchical organization legitimized by social *functions* wielded by the chief for common benefit (according to Service mostly functions of a redistributive nature) in a theocratic frame of reference, where »economic and political functions were all overlaid or subsumed by the priestly aspects of the organization« [*ibid.,* 305].

Another oft-quoted contributor to the general debate should be singled out for relevance for the following. Robert Carneiro, arguing (1981:58) that »what a chief gets from redistribution proper is esteem, not power«, observes [*ibid.,* 61] that

(C) As long as a chief merely returns everything he has been handed, he gains nothing in wealth or power. Only when he begins to keep a large part of it, sharing with his retainers and supporters but not beyond that, does his power begin to augment.

But the power of a chief to appropriate and retain food does not flow automatically from his right to collect and redistribute it. Villagers freely allow a chief to equalize each family's share of meat or fish or crops through redistribution because they benefit from it. But they will not willingly suffer the same chief to keep the lion's share of food for himself. Before doing this, he must acquire additional power, and that power must come from some other source.

Power, then, depends on the ability of the chief to transform redistribution proper (where the chief retains only a small percentage of what passes through his hands) into *tribute* or *taxation*, where he keeps a large part for himself and for the »core of officials, warriors, henchmen, retainers, and the like who will be personally loyal to him and through whom he can issue orders and have them obeyed« [*ibid.*, 61]. The origin of this transformation Carneiro sees in *warfare resulting from population pressure*. Warfare is the reason that early class societies consist of *three* and not just two classes [*ibid.*, 65]:

(D) The two classes that are added to a society as it develops are a lower class and an upper class, and the rise of these two classes is closely interrelated. The lower class [...] consists initially of prisoners who are turned into slaves and servants. At the same time, however, an upper class also emerges, because those who capture and keep slaves, or have slaves bestowed upon them, gain wealth, prestige, leisure and power through being able to command the labor of these slaves.

Even though considering the transition »from autonomous villages, through chiefdoms and states, to empires« as a continuous process [*ibid.*, 67], Carneiro finally finds it useful to distinguish the state [*ibid.*, 69, quoting *idem* 1970:733] as

(E) an autonomous political unit, encompassing many communities within its territory and having a centralized government with the power to draft men for war or work, levy and collect taxes, and decree and enforce laws.

Though illustrated by references to ethnographic and historical material, the theories cited here are *general theories*. During the last 15 to 20 years they have been tried out by specialists on a large number of single cases, which has provided many insights into the applicability of the concepts involved and into the historical variability of the diverse processes to which the theories make appeal. It would lead too far to discuss them in general[3], and I shall only quote two of special relevance for the Mesopotamian case. Firstly, in a discussion of Archaic Greece Runciman [1982:351] distinguishes »the emergence of a state from nonstate or stateless forms of social organization« by »these necessary and jointly sufficient criteria«:

(F) Specialization of governmental roles; centralization of enforceable authority; permanence, or at least more than ephemeral stability, of structure; and emancipation from real or fictive kinship as the basis of relations between the occupants of governmental roles and those whom they govern.

Secondly, working on Mesopotamian and Iranian material Henry T. Wright and Gregory A. Johnson [1975:267] formulate a description focusing »on the total organization of decision-making activities rather than on any list of criteria«, defining a state

(G) as a society with specialized administrative activities. By »administrative« we mean »control«, thus including what is commonly termed »politics« under administration. In states as defined for purposes of this study, decision-making activities are differentiated or specialized in two ways. First, there is a hierarchy of control in which the highest level involves making decisions about other, lower-order decisions rather than about any particular condition or movement of material goods or people. Any society with three or more levels of decision-making hierarchy must necessarily involve such specialization because the lowest or first-order decision-making will be directly involved in productive and transfer activities and second-order decision-making will be coordinating these and correcting their material errors. However, third-order decision-making will be concerned with coordinating and correcting these corrections. Second, the effectiveness of such a hierarchy of control is facilitated by the complementary specialization of information processing activities into observing, summarizing, message-carrying, data-storing, and actual decision-making. This both enables the efficient handling of masses of information and decisions moving through a control hierarchy with three or more levels, and undercuts the independence of subordinates.

Unless »information«, »data-storing« etc. are taken in a rather loose sense, societies traditionally regarded as indubitable states (like Charlemagne's Empire) may well fall outside this definition. But in the Irano-

[3] A large number of case studies and further references will be found in [Claessen & Skalník 1978] and in [Gledhill, Bender & Larsen 1988].

Mesopotamian case the authors succeed in making it operational by means of sophisticated archaeology and through the application of geographical »central place theory«. Furthermore, the specific definition of »control« involved may serve to distinguish the specific character of Irano-Mesopotamian state formation.

Control, indeed, may differ in kind—even control developed to the degree of vertical and horizontal specialization and division of labour described by Wright & Johnson. But if control and decision-making involve intense message-carrying and *data*-storing as the fundament for further decision-making, as was the case in Mesopotamia (cf. below), then some means for accounting and the handling of data must develop together with the state—be it writing and numerical notations, be it something like the Andean *quipu*, be it some third possibility. For this same reason, indeed, »archaeologists like[d] to use "writing" as a criterion of civilization« (roughly synonymous with statal culture), as Gordon Childe pointed out in 1950[4], while at the same time himself pointing to the equally important role of accounting [*ibid.*, 14]. This brings us back to the problem of Mesopotamia.

III. The rise of states in Southern Mesopotamia

The centre of early Mesopotamian state formation was the southernmost part of Mesopotamia (»Sumer«); furthermore, for the whole period which I am going to consider in depth, the essential developments as far as mathematics is concerned took place in the Sumerian and Babylonian South-to-centre—whence the above caption. A description of the pre-historic development, however, cannot be circumscribed meaningfully to this area—already because most of the Sumerian territory was covered by water during the larger part of the prehistoric period, but also because much

[4] [Childe 1950:3]. A recent comprehensive discussion of the connection between state formation, writing and alternatives to writing is [M. Tr. Larsen 1988].

wider areas were involved in parallel developments.

By 8000 B.C., permanent settlements had been established and agriculture and herding had become the principal modes of subsistence, although hunting, food-gathering and fishing remained important subsidiaries well into historical times. Within the single settlements, social stratification may have developed around redistribution—needed precisely because of the combination of several complementary subsistence modes, cf. quotation (C). The single villages, however, were involved in no higher structures of settlement or redistribution—their very ecological localization shows that they were meant to live on their own, apart from participation in long distance trade in obsidian and similar scarce goods. This self-sufficiency holds good even for the rare large settlements like Jericho, level B (7th millennium), with at least 2000 inhabitants, and Çatal Hüyük (6th millennium) with at least 5000, although the internal social organization and stratification will probably have been much more complex here than in smaller settlements[5].

In the sixth to fifth millennium, the paths followed by different parts of the Middle East diverged. In geographically suitable places like the Susiana plain in Khuzestan (Southwestern Iran), larger numbers of settlements can be seen to form interconnected systems, some of them possessing apparently central functions or positions (to judge, *inter alia*, from systematic size differentiations)[6]. In the late fifth millennium, the city Susa had an area of some 10 hectares and was the centre of a system of smaller settlements in Susiana. Central store rooms in what may be a sacred domain have been found in the city, and findings of seals and seal impressions in Susa and a neighbouring small settlement bear witness of controlled delivery of goods from the small settlement to the centre. But no traces of higher level recording or summarizing occur in this archaeological layer [Wright & Johnson 1975:273].

After a setback in population density[7], the »Early Uruk« period (before

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[5] Cf. for this description [Nissen 1983:36-40, 55]; [Mellaart 1978]; and below.

[6] See [Wright & Johnson 1975:269f], and [Nissen 1983:57f].

[7] Disputed by Weiss [1977]. The difference of opinion depends on different estimates for the relative lengths of archaeological periods, again dependent on different

the mid-fourth millennium) brought new expansion. Susa had grown to 13 hectares and was the centre of a three-level settlement system, ceramic ware was distributed from central workshops, and bitumen, chert and alabaster were produced locally for exchange. In the following (»Middle Uruk«) period, the size of Susa doubled to 25 hectares and the city was internally differentiated; the settlement system became four-tiered, there is direct evidence for differentiated levels of administrative control (by means of seals, »tokens« and »bullae«, cf. below), and perhaps already indirect evidence for the distribution of standardized grain rations to institution workers[8]. In the Late Uruk period (3500-3100 B.C.), the trend toward specialization and hierarchical control continued. Now, however, a similar level was reached in Southern Mesopotamia, where Uruk became the dominant centre. Susa, on the other hand, fell behind, and will be less interesting for the arguments of the following[9].

The reason for this development is to be sought in climatic changes, which lowered the water-level in the Gulf by some 3 m after the mid-fourth-millennium and diminished the rainfall in the area [Nissen 1983:58-60]. As a consequence, land which had been covered by salt marshes or had been inundated regularly by the rivers now became available for irrigation agriculture. Until then, settlements in Southern Mesopotamia had been rather few and not part of higher-level systems. Now, however,

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absolute datings. The most recent calibrated radiocarbon datings appear to favour Wright and Johnson [B. D. Hermansen, personal communication].

[8] See [Wright & Johnson 1975:272, 282f] and [Johnson 1975:295-306]. The presumed evidence for ration distribution (the particular »bevelled rim bowl«) has been challenged by Beale [1978]. In proto-literate Uruk (see below), however, the connection between the bowl in question and the delivery of rations is corroborated by its seeming appearance in the pictogram for rations (KU_2). Cf. also [Damerow & Englund 1989:26].

[9] Writing turns up in Susa (and indeed in the Iranian area at large) somewhat after its emergence in Uruk. The idea of writing appears to be borrowed, but the pictographic script itself is independent—while, on the other hand, there is clear kinship but not identity between the »proto-Sumerian« and the Iranian »proto-Elamite« counting and metrological systems. For detailed information I shall only refer to [Damerow & Englund 1989] (including Lamberg-Karlovsky's introduction to that work).

a larger settlement density (larger than anything known before in the Near East) and the creation of a noticeable surplus in agriculture became possible. The city Uruk (as large as 50 hectares in the Late Uruk period, and soon much larger still) became the centre of a 4-tiered settlement structure; the internal productive and administrative organization of the city was highly differentiated, vertically as well as horizontally; huge public works in the form of temple building were performed, workers as well as officials being paid in rations in kind; and outposts in Northern Iraq as well as trading relations to Bahrain were established[10].

The evidence for this development is two-fold. Part of it is made up by the traditional archaeologists' array of settlement and building remains and of other artefacts. Part of it, however, consists of *carriers of meaning:* pictures carved in cylinder seals, on relief vases, etc.; and inscribed clay tablets, first with numbers only (in the pre-literate »Uruk V« stratum, before 3300 B.C.) and then also with pictographic writing (in the »proto-literate« Uruk IV and Uruk III periods, 3300-2900 B.C.).

Even though there is an indubitable continuity from the Late Uruk script to the later Sumerian cuneiform, it is far from completely deciphered; cylinder seals, like all other pictures, are of course always ambiguous and polyvalent. None the less, the combination of these carriers of social and linguistic meaning (and more than that, cf. below) conveys a lot of information not available from earlier periods. Prominent facets of the picture which emerges are these:

— The city (and, as a consequence, the settlement system whose centre it was) was under theocratic control. Its core was made up by a sacred terrain dominated by a number of large temples, which can only have been built because of the existence (and availability to the theocratic rulers) of a large agricultural surplus.
— Part of this surplus was apparently given as tribute—a famous temple vase shows a procession bringing offerings to the city Goddess Inanna (reproduction and discussion in [Nissen 1983:113-115]). But part of it must also have been extracted from labourers working directly on

[10] [Adams & Nissen 1972:17-19]; [Johnson 1975:310-324]; [Nissen 1983:73-116, 132-134; 1986a:330].

Temple domains, many of them most likely enslaved prisoners—apparently the most popular theme of the cylinder seals of high Temple officials shows vanquished and pinioned prisoners watched by a high (supreme?) official and being beaten up more or less explicitly (reproduction of select specimens and discussion in [Nissen 1986:146-148]).

— The ruling group of the city was constituted by the top officials in a hierarchy also encompassing lower officials and craftsmen's and workers' foremen (cf. below, on the »profession list«). All appear to have received rations in kind in some sort of quasi-redistributive system, while at least the highest officials received important allotments of land ([Vaiman 1974:20f]; whether this land was worked by personal servants or slaves or by »public« labourers is unclear).

— Quasi- (or pseudo-)redistributive features were also furthered by the lack of virtually all natural ressources apart from pastures, agricultural land, fish, fowl, reed and clay. All needs apart from these (in particular, i.e., those arising from temple building and equipment and the luxury needs of the governing group) depended on organized import and distribution.

— To keep track of tribute and other deliveries and of the products of public agriculture and herding, and also in order to calculate the rations of officials, workmen and domestic animals, techniques for accounting and computation were developed (details below). In the earliest (»Uruk V«) phase, tablets carrying only numerical / metrological inscriptions and seal impressions of responsible officials were employed. Whether used for accounting, as receipts or as delivery notes they could only be understood by somebody possessing full knowledge of the context of the transaction in question. In the next, terminal phase of the »Late Uruk« period (stratum IV, 3300-3100 B.C.), pictograms are put together with the numbers. Even though there is no full rendition of any spoken language, nor any attempt to render syntax, the tablets could now be used as supports for memory, and to summarize a whole series of transactions while tracking its course—especially because the tablets are written according to a fixed format for single transactions and totals. In the ensuing »Uruk III« or »Jemdet Nasr« phase (c. 3100 to c. 2900

B.C.), these formats grow more complex and more regular[11].

— There is no doubt that the script was developed as an accounting and control device. 85% of all written documents belong to the category of economic tablets. The remaining 15% are made up by »lexical lists«, apparently used for teaching purposes. A »profession list« describing the hierarchy of officials and professions turns up most frequently in the record. Other lists enumerate herbs; trees and wooden objects; dogs; fish; cattle; birds; place-names; vessel-types; and metal objects (see [Nissen 1981]). Literary and religious texts are as absent as monumental inscriptions.

— Nothing in the record suggests that general Temple functions, management of the Temple estate and practical book-keeping were separated. To the contrary, literacy (confined to the sole function of economic control) will probably have been too restricted for any full separation to have taken place (nor has a specific scribal function been identified in the profession list). As to the merging of priestly functions and Temple estate management, precisely the sanctification of originally redistributive functions will have made possible that transformation of redistribution into taxation which might otherwise have been impossible (cf. quotation (C)).

While this much is fairly well-established, other questions remain open—not only because the script is largely undeciphered but also because of the nature of the written evidence. Three open questions are of some relevance for the present study.

First of all, the reach of statal domination is unclear. The profession list as well as the location and immense size of temple buildings tells us that the statal institution *par excellence*, irrespective of our choice of precise

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[11] The tablets are never found in the places where they were originally made or used but mostly in rubbish heaps. The relative dating thus relies on paleographic criteria, which, however, seem reliable (see [Nissen 1986a:319-322] for details). Because of the greater complexity and regularity of Uruk III tablets, some of the administrative features ascribed here to the whole proto-literate period may indeed only be fully developed in the later phase.

The organization of text formats and the use of formats as carriers of information are explained and discussed in [Green 1981:348-356].

defining criteria, was the Temple. We know that the Temple bureaucracy had command of a large work force, that these workers as well as a number of officials of varying rank were supplied rations in kind. But we do not know how many of the workers were enslaved, nor whether there existed a stratum of peasants only loosely submitted to the Temple (paying, e.g., a limited tribute in form of temple offerings or perhaps none at all, maybe and maybe not contributing corvée labour)[12]. Temple accountants, after all, recorded transactions which regarded the Temple economy; they were not engaged in social statistics. Evidence from the third millennium suggests that free, kin-based peasant communities will have been an important part of the total social fabric[13].

Secondly, we do not know the real constitution of the bureaucracy. Because we only know it from accounting and glyptics we may be inclined to see it as a suppressive and theocratic yet fundamentally Weberian bureaucracy. Ethnographically, however, this picture is highly improbable, and prosopographic studies of third millennium material has given Marvin Powell [1986:10] the impression that »the entire bureaucracy is knit together by an elaborate system of kinship, i.e., what we would call nepotism and influence«.

Thirdly, the specific organization of urban society, of the total settlement structure (not least concerning outposts like the town Habuba Kabira built in Northern Mesopotamia during Uruk V and then abandoned, and the relations to other administrative centres developing no later than Uruk III) and of most trades and handicrafts is unclear. Were traders Temple officials (in the mid-third millennium, some private venture must be presumed, see [Adams 1974:248])? Were the »chief«, »junior chiefs« and »foremen« of the professions testified in the profession list (see [Nissen

[12] Details of the settlement structure, it is true, suggest that an inner core of settlements (until some 12 km from the city) was bound more strongly to the centre than those farther removed [Nissen 1983:144f]. The outer zone can be surmised not to belong to the Temple estate proper; but we have no means to assure that all land of the inner zone was submitted uniformly to the theocratic system.

[13] For a discussion of the general arguments for the presence of such communities, see [Diakonoff 1975]. [Diakonoff 1969a] is an English summary of his epoch-making investigation of 24th century Lagaš.

1974:12-14]) really members of an all-encompassing hierarchy, or is the organization of the profession list due to the particular and biased perspective of literate Temple bureaucrats? Is the appearance of the »chairman of the assembly« in the profession list an indication that a formerly primitive-democratic assembly of citizens had been subsumed under the Temple hierarchy, or is this just an expression of priestly wishful thinking[14]? Once again, third millennium parallels suggest that the real situation was more intricate than the information which we are able to extract from the written documents.

These conclusions from third millennium parallels may be combined with an observation made by Joan Oates [1960:44-46]: since both the essentials of temple groundplans in Eridu (one of the originally isolated settlements of the extreme south) and many other religious customs exhibit continuity since the fifth millennium, at least the culturally pivotal segment of the Late Uruk state building population appears to be autochthonous. The violent increase in population after the mid-fourth millennium, on the other hand, is probably not to be ascribed to autochthonous breeding alone. Influx of new population segments regimented somehow by the Temple institution (whose organization may have taken over much from the corresponding Susa institution) may have contributed to the creation of the three-class situation described by Carneiro (see quotation (D)): thanks to the surplus extracted from the Temple clients and subjects, the Temple staff could evolve into a new upper class, while the clients and possible enslaved workers made up the new lower class. Non-subject populations (be they autochthonous or immigrants) may have continued a traditional non-state existence with only limited submission to the statal institution[15].

[14] Or ours? Our own bureaucratic conditioning in combination with the internal rationality of the book-keeping records may easily lead us into more Weberian readings of the text than intended by its original authors.

[15] While proto-literate Uruk was a full-fledged state according to Wright and Johnson (quotation G) it is thus far from certain that it would be so according to Carneiro (quotation E) and Runciman (quotation F). From their point of view, the control system will probably have directed not *a state* but only *an estate* immersed into and influencing a pre-state society. Especially for Runciman, who sees early seventh century Athens as a »proto-state« only, the proto-literate Uruk system can have

For the very same reason, however, they will have been out of the administrative focus of the Temple managers. That accounting rationality which, as we shall see, contributed to the formation of mathematics, was only concerned with the relation between the Temple estate and its officials and dependants—and whatever the real complexities of state formation, the written record only reflects the pseudo-redistributive features of the situation.

As long as we restrict ourselves to the proto-literate period alone, however, all talk of the »real complexities« is, firstly, pure speculation, and secondly inane speculation. It is only given sense by the perspective of the following, »Early Dynastic« period (cf. [Diakonoff 1969a:178-180], and [Powell 1978:139]).

IV. City states and centralization

Apart from an initial lacuna of some 200 years in the written record, the source situation improves steadily and significantly during the following millennium. This has several reasons.

— Firstly, the script evolved to the point where it is fairly well understood—both because of changes in the sign repertoire and because of incipient use of syllabic writing. Due to the latter development we even know that the language in use was now Sumerian, while we have no means to decide in which language the pictographs of the proto-literate period were told[16].

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been no more.

[16] Traditionally, it is true, the opposite view has been accepted on preliminary evidence from a single, somewhat ambiguous sign combination in a single text. However, the ongoing progress on a large project on the archaic texts directed by Hans Nissen (see [Nissen 1986b]; the results of the project are reflected in many references in the present paper) has uncovered no supplementary testimony; for this and other reasons discussed by Robert Englund [1988:131-33] in a two-page footnote, we must now opt for a vigorous *nescimus*.

— Secondly, writing was used much more broadly and more systematically. Around the mid-third millennium royal inscriptions, literary texts and political and juridical documents (some of them involving communal and private land) turn up. Even the traditional genre, the economic texts, improves in coherence and systematization.
— thirdly, certain aspects of early third millennium society are reflected in oral epics written down in the second half of the millennium.
— fourthly and finally, on a number of archaeological sites strata from the third millennium cover those from the late proto-literate period, for which reason the latter are poorly known.

The first 500 years after the proto-literate phase are known as the Early Dynastic period (ED). Its first part is characterized by continued population growth—around 2900 B.C. Uruk had grown to 6 km^2, half of Imperial Rome at its culmination—and by further diminishing rainfall and lowering water-level in the Gulf and hence also in the great rivers. Around the mid-third millennium, moreover, a new main branch of the Euphrates was formed. This had decisive consequences, as discussed in some detail by Nissen [1983:141-148]. What is important in the present connection is the development of a system of city states, competing and often at war for the same water resources; and of kingly functions in these city states, formally originating as Temple offices but in reality regents on their own and eager to stand forward in their inscriptions as benefactors and protectors of the temples of their cities and city gods (see the collection of royal inscriptions in [Sollberger & Kupper 1971]).

One of the Sumerian epics offers an interesting insight into the social structure, somewhat at cross purposes with naive identification of State and Temple estate. In *Gilgameš and Agga* (translated in [Römer 1980]) we are told that Agga, son of king Enmebaraggesi, proceeds with his army from Kish to Uruk and delivers an ultimatum. King Gilgameš of Uruk first tries to convince the council of elders of his city to fight back; he fails, and

My present pet hypothesis (the reasons for which I present in [Høyrup 1992]) is that Sumerian shares so many grammatical features with »creole languages« (on which see, e.g., [Romaine 1988]) that it may have originated as a creole at the influx of new population segments in the later fourth millennium.

instead he puts the matter before the council of »men« (capable of bearing arms? or commoners, if the »elders« are elder by status and not by age?), who agree with Gilgameš and entreat the aristocrats and mighty of the city to fight for Eanna, the city's temple established by An the heavenly god and »cared for« by the hero-king.

Most likely, the epic was only committed to writing toward the end of the third millennium; but since Enmebaragesi is a historical person (he has left an inscription, and belongs around the 27th century, in early ED II) the written text must build on fairly stable oral transmissions. Moreover, the conciliar institutions were definitely not as powerful toward the end of the millennium as presupposed by the text. The social situation delineated in the poem must therefore correspond to *some* historical reality.

That, however, is striking. Admittedly, Eanna is mentioned as the pride of the city—but definitely not as supreme owner or overlord. The affairs of the city are taken care of by the king in agreement with the two councils. The whole make-up reminds more of the *Iliad* than of the managerial society intimated by the proto-literate archives. If the higher Temple officials are mentioned (and they probably are!) it is only as rich and powerful »1st class citizens«, i.e., as aristocrats or »elders«.

On the other hand, there is no doubt that the managerial tradition *was* very much alive, as testified by the continued and expanding use of the same script and the same accounting techniques as in Uruk IV-III, and by the persistent use of the familiar lexical lists. We are thus confronted with a truly *dual society*, as suggested above: one aspect can be described with some approximation as that »military democracy« which Engels portrays in *Der Ursprung der Familie*[17]. The other is the formally redistributive, functional state presupposed in his *Anti-Dühring*—and since these two complementary theories anticipate the main approaches of modern political anthropology we may conclude that the disagreements within this field correspond to the dialectic of real state formation[18].

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[17] This aspect has been investigated by Thorkild Jacobsen in several publications [1943; 1957].

[18] Basing himself on other evidence, Nissen [1982] argues for duality of the Sumerian society along several other dimensions.

... At least to *the dialectic of real state formation as it happened in Mesopotamia*. The duality is, indeed, more obvious here than in many other cases (cf. however chapter XII on parallels in Medieval Europe). That is seen, e.g., if one compares the ways in which early Mesopotamian and other ancient monarchs made use of the techniques of literacy, once developed for accounting, to glorify themselves. While most royal inscriptions of the Ancient world boast of prowess and military success, until mid-ED III Sumerian royal inscriptions boast of temple building, of gifts given to the temple, of ceremonies performed, and of canal-building. Early Mesopotamian literacy was thus no transparent medium but a strong ideological filter which would not allow certain utmost important aspects of the kingly function to be seen[19].

Towards the end of the Early Dynastic period the temples and temple estates have come under the sway of the city kings, who treat them as their private property[20]. The existence of communal land is testified by sales contracts, but these always show that the land is sold to private individuals with high social status (high officials, members of the royal family), and often »at a nominal price« ([Diakonoff 1969a:177]; cf. [Powell 1978:136f]). Since peasant clans will in any case only have sold their hereditary land when in distress or when submitted to severe pressure, we may conclude that this was probably the point where a state in Runciman's sense was

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[19] In this connection one may also recall the oft-made observation that nobody would have guessed from the written record that Sumerian rulers might be buried with a large retinue of killed servants (as it was actually the case in Ur, during the first phase of ED III).

[20] This is particularly clear in a series of »reform texts« by Uru'inimgina, either elected king of Lagaš by the assembly or usurper in the late 24th century B.C., describing the abuses which had developed and his restoration of good old time, which includes giving back the temple land appropriated by the ruler to the gods (a recent though not fully convincing discussion of the obscure texts and an exhaustive bibliography is [Foster 1981:230-237]; cf. also [Hruska 1973]). But since Uru'inimgina and his consort are to function as stewards of the gods on their reacquired estates, realities did not change at least on this point [Tyumenev 1969a:93f]. Whether his protection of »widows and orphans« fared any better is unclear. In any case, Uru'inimgina was soon brushed aside by Lugalzagesi's conquest and unification of the whole Sumerian region.

establishing itself (cf. above, quotation F, and note 15).

In the mid-24th century, as a next developmental step, the whole Sumerian region was then united into one territorial state by conquering kings, first by Lugalzagesi of Umma and soon afterwards by the Semitic Sargon of Akkad. Powell [1978] sees this as a result of the conflicts arising, *inter alia*, from growing social and political tensions caused by the increase of private large-scale property—tensions which could not be released or held in check within the single city state, in spite of attempts like Uru'inimgina's »social reform«[21]. From now on, the »despotic« territorial state or empire can be regarded as a rule in Mesopotamia and the decentralized phases as interludes.

For reasons of obvious necessity, Sargon and his dynasty introduced more far-ranging social controls than any predecessor, many of them further developments of the traditional accounting controls. Already the Early Dynastic radical transformations of the socio-political structure, however, had led to changes in the domain of written administration. Both phases of the transformation were reflected in the structure and practices of the environment responsible for this administration. The evolution which took place during the Sargonic reign continued trends established during the preceding two centuries while at the same time reshaping them to the advantage of government.

The first step is testified in Fara (Ancient Šuruppak) around the mid-third millennium. Here, for the first time, *the scribes* turn up in the administrative documents as a separate and hierarchically organized group, even provided with overseers and a »senior scribe« [Tyumenev 1969:77]; until then, the very term is absent from the sources—with the exception of one Jemdet-Nasr tablet which shows that the profession is not hidden in one

[21] In fact, the analysis reminds strikingly of Engels' (and Aristotle's) analysis of the Solon reforms in *Die Ursprung der Familie ...* . Even this formation of a mature state in Athens followed upon a phase considered as »military democracy«—and followed shortly after the establishment of a state in Runciman's sense.

That conflicts between the city states became intense in late ED III is obvious from the surviving royal inscriptions. After centuries of mounting city-walls combined with amazing royal taciturnity on warlike matters, proclamations of military triumph and menaces against potential aggressors suddenly abound.

of the uninterpreted lines of the proto-literate profession list.

The reason for the emergence of the profession is probably straightforward: writing itself was used more widely for socially important purposes, apparently in connection with the beginning of the above-mentioned socio-economic transformations of ED III (see [Powell 1978:136f]). It is precisely in Fara that legal contracts, *viz* concerning the sale of land, turn up (see [Krecher 1973, 1974]). In Fara, too, a monetary function becomes visible for the first time (in Fara accomplished by copper, in later ED III by silver). Temple estate accounting, too, grew in extent and systematics. We seem to stand at the threshold dividing »ultra-limited literacy« from »limited literacy«, to use a conceptual distinction proposed by John Baines [1988: 208].

As pointed out by Baines, »limited literacy« is really a *new* situation, with problems and possibilities of its own. First of all this reflects itself in the education of the literate-to-be. Even though the old lexical lists were still in full use (but in decline after Fara), new types of school-texts emerge, as it appears from Deimel's collection ([1923]; on p. 63 we find a student's drawing of the proud teacher); of special importance are the mathematical exercises, to which I shall return below. Finally, the Fara period produced the beginning of *literary texts*, testified by fragments of a temple hymn and by the first proverb collection [Alster 1975:15, 110]. It seems that the scribes, once they had become a profession halfway on their own[22], tried out the possibilities of the professional tools beyond their traditional scope (this will be even more obvious when we come to the mathematical exercises)—and a perusal of the tablets which the Fara scribe students produced suggests that they liked the enterprise: in many of the empty corners of tablets, irrelevant but nice drawings have been made, portraying teachers or deers or featuring complex geometrical patterns. One gets an immediate impression of enthusiasm for the freshness of scholarly activities similar to that reported from Charlemagne's Palace School in Aachen or from Abaelard's and Hugue of Saint Victor's »12th century renaissance«.

[22] Halfway only—many of Deimel's didactical tablets carry names of what seems to be authors, editors or teachers, and many of the persons mentioned carry a priestly title [Deimel 1923:2*f].

The trends beginning in Fara continue during ED III, during the Sargonic era, and during the post-Sargonic decentralized 22nd century interlude. The number of legal contracts of many sorts keeps growing; archives are used on many levels[23]. Systematic school teaching continues, though relatively few records (among which, however, mathematical exercises) survive. Writing becomes more phonetic and orderly already in ED III (Maurice Lambert [1952:76] speaks of an outright *reform of writing* under Eannatum of Lagaš)[24]. Even the creation of literary text continues, though with a change. No longer an expression of semi-autonomous scribal identity, hymns are written in the royal environment where they serve to demonstrate the king's affection for those temple institutions which had been subjected to royal authority, as discussed by William Hallo [1976:184-186]. Sargon's daughter Enḫeduanna may indeed be the first poet in world history known by name. Gudea, the most important ruler of Lagaš during the post-Sargonic decentralized interlude, appears to have had epics composed on command which transposed his own feats into the mythical past. Also in another respect is he seen parading as a culture hero: not only a temple builder in the abstract like the kings of earlier inscriptions, Gudea has drafted the ground-plan himself »in the likeness of Nisaba [the scribal goddess], who knows the essence of counting« (Cylinder A, **19**, 20-21, in [Thureau-Dangin 1907:110]); he has also formed and baked the brick, brought precious materials from foreign countries, and performed all other crucial steps in person. Though the ruler of a city-state similar to those of former times and perhaps conscious of himself as a restorator of the order of old, Gudea no less than the Sargonides represents the tendency to make *inter alia* scribal culture subservient to a fundamentally secular power.

This is no less true in the following centralizing period, the so-called Third Dynasty of Ur or »Ur III« (not to be mixed up with »Uruk III«, a period named after an archaeological stratum in a different city), coinciding

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[23] Foster [1982:7-11] distinguishes three Sargonic archive types: family or private; »household« (with a horizon restricted to a single city) and »large household«.

[24] When systematic writing of the Semitic Akkadian began, using the phonetic values of the Sumerian signs, orderly succession of the signs became compulsory.

with the 21st century B.C.[25] The founding king, Urnammu, subdued the whole of Southern Iraq, and undertook large building programs. Since relatively few written documents are known from his time, we have no detailed knowledge of his policies, nor from the first 20 years of his successor Šulgi. At that point, however, Šulgi instituted a military and administrative reform, and from then on huge amounts of administrative tablets exist. They uncover a centralized economy submitted to meticulous control. It is probably *not* true, as has been believed, that *all* land belonged to the state or to temple estates in practice controlled by the state; that *all* industry was governmental; that *all* merchants were *exclusively* government agents; nor that *all* manual work was done by semi-enslaved populations. But the very fact that these theses have been widely held show that royal estates, governmental trade and governmental workshops and even textile factories worked by slaves were all-important[26]. The precise booking of rations, work-days, and of flight, illness and death within the work-force allotted to each overseer also reveals an extremely harsh regime. As pointed out by Robert Englund in his conclusive words [1990:316], the understanding of working conditions conveyed by the administrative texts »kann vielleicht helfen, sich in den historischen Darstellungen des 3. Jahrtausends v. Chr. die Kosten der babylonischen Paläste und Statuen plastischer vorzustellen«.

In this situation, whatever autonomy may have been left to communities and crafts will have been severely restricted. This is demonstrably true for scribal culture. The scribe, of course, was the pivot and, in principle, the hero of an administrative system the precision and scope of which Nikolaus Schneider [1940:4] regarded as »überspitzt« even from his writing perspective within the National Socialist war economy. The scribal title was used as an honorific title of dignitaries in general [Falkenstein 1953:128]. Moreover, in one of the hymns glorifying King Šulgi he also

[25] Brief expositions are given by Nissen [1983:207-213] and Liverani [1988:267-283]. A recent critical survey of the state of the art concerning Ur III administration is given by Robert Englund [1990:1-6].

[26] An overview of the centralized economy as well as the exceptions is given by Hans Neumann [1988]. Cf. also [Neumann 1987:151-154] on non-statal artisanate.

presents himself as »a wise scribe of [the scribal goddess] Nisaba«, a characteristic which stands as the culmination of a long series of images (transl. [Klein 1981:189, 191]):

(H) I, the king, from the womb I am a hero, [...], I am a fierce-faced lion, begotten by a dragon, [...], I am the noble one, the god of all the lands, [...], I am the man whose fate was decreed by Enlil, [...], I am Šulgi who was voluptuously chosen by Inanna [goddess of Uruk], I am a horse, waving its tail on the highway, [...], I am a wise scribe of Nisaba. Like my heroism, like my strength, my wisdom is perfected, its true words I attain, righteousness I cherish, falsehood I do not tolerate, words of fraud I hate!

Looking back at *Gilgameš and Agga* we observe that nothing is left of dual society. The world of kingly prowess and that of scribal administration (identified with wisdom and justice) are united in the same person who boasts on both accounts in the same composition.

The so-called Ur-Nammu law-code, which should in fact carry Šulgi's name ([Kramer 1983]; cf. [Neumann 1989]), shows a similar mixture in its prologue (ed., transl. [Finkelstein 1969:66-68]). At the same time it elucidates the royal idea of justice, which on one hand involves metrological regularization and reform, on the other repeats the nice words (and the details!) of Uru'inimgina and Gudea too much in the manner of a literary *topos* to be really convincing (cf. [Edzard 1974]).

Two other Šulgi hymns [Sjöberg 1976:172f] tell about the king's purported time in the scribal school, and thus make clear which aspects of scribal cunning were central seen from the official perspective (which, we can be fairly sure in a society like Ur III, was also the perspective communicated to the students): addition, subtraction, counting and accounting according to one; writing, field-mensuration and drawing of plans, agriculture, counting and accounting (and a couple of ill-understood subjects) to the other.

Traditional *topoi* and nice hand-writing apart, the idea of *justice* had been reduced to unified metrology and menaces against trespassers of royal regulations, and that of scribal art to functionality within the administrative apparatus. According to all evidence, scribes were taught in school to be proud of their function in the administrative machinery; no more place is left (in the official ideal) to professional autonomy than to communal primitive democracy. The higher level of literary (and, as we shall see below, mathematical) creativity was in all probability the preserve of a »court chancellery« (»Hofkanzlei«, [Kraus 1973:23]) where year names, royal hymns, politically suitable epic poems and royal inscriptions were produced. On all accounts, the scribal art had been harnessed to a no longer dual state—in trite practice in as far as rank-and-file scribes are concerned, as a source for ideology in the case of the elite.

V. Breakdown and apogee

In spite of the immense role played by the scribes in Ur III, the problems associated with »limited literacy« appear to have been solved or suppressed. Scribal autonomous thought, as any autonomy except perhaps nepotism and appropriation of »public« property among the privileged, is absent from the sources. But the cost of bureaucratic control was too high, and the price of extensive building activities and an all-encompassing administrative network was a work-force plagued by illness, death and problems of flight—and even, if we are to believe indirect literary evidence, rebellious strikes[27]. Internal breakdown resulted[28],

[27] After toiling 40 years night and day in the great marsh, the minor gods decide to confront their chamberlain (the god Enlil); they do so, armed with spades and hods to which they have set fire, and claim that the chamberlain call in the collective leadership (consisting of Enlil himself together with the gods An and Enki). When asked for the instigator, the strikers deny the existence of such a person and declare their solidarity—thus begins the plot of the Old Babylonian *Story of Atraḫasīs* (ed., tr. [W. G. Lambert & A. R. Millard 1969]; this passage pp. 45ff). The whole description is too close to the social psychology of real wild cat strikes to have been freely invented, and the setting suggests that the author builds on experience from Ur III estates rather than contemporary events.

In the end, the problem is solved by a »social reform«: *man* is created in order to take over the toil of the gods.

[28] Most likely, ecological reasons were also involved in the breakdown, accentuating the incompatibility between the costs of the state apparatus and the productivity of the work force. In any case, the political centre of Iraq from now on moved

followed by now irresistible barbarian invasions and another interlude of decentralization, the beginning of the »Old Babylonian« period (2000 to 1600 B.C.).

One of the resulting smaller states (Isin) continued the Ur III system as best it could for a century, and has provided us with a school hymn describing the high points of the scribal art as embellished »writing on the tablets« together with use of »the measuring rod, the gleaming surveyor's line, the cubit ruler which gives wisdom«[29], not far from Šulgi's ideals though without his emphasis on accounting. The other main successor state (Larsa) inaugurated a trend which was to culminate during the next phase of centralization, achieved by Hammurapi of Babylon (1793-1750)[30]. On the whole, the system of state-controlled production was abandoned. Royal land was often (though not always) given to tenants instead of being organized as large estates run by servile labour, or it was assigned to officials or soldiers who leased it to farmers. Similarly, land belonging to wealthy city-dwellers was often leased—and in general, private possession of large-scale landed property became common. (The survival of community-owned land is disputed, cf. [Komoróczy 1978] versus [Diakonoff 1971]).

Similarly, public foreign trade was replaced by private trade; at least one major city appears to have been run by the body of merchants with some autonomy [Oppenheim 1967]. Royal workshops had probably been taken over by their managers at the breakdown of the Ur III system, and were now run privately; free labourers working for wage largely replaced the semi-enslaved workers receiving rations in kind. We even observe a kind of banking developing, conducted by members of an institution for unmarried noble-class women using their double kinship affiliation (to the

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northwards.

[29] Lipit-eštar Hymn B, lines 21-23, transl. [Vanstiphout 1978:37].

[30] A very readable narrative not only of Hammurapi's history and policies but also of the socio-political and cultural conditions since early Old Babylonian times is given by Horst Klengel [1980]. Other works to be consulted include [Dandamajev 1971], [Diakonoff 1971], [Gelb 1965], [Jakobson 1971], [Klengel 1974, 1977], [Komoróczy 1978, 1979], [Kraus 1973], [Leemans 1950], [Oppenheim 1967], [Renger 1979], and [Stone 1982].

real kin, and to the pseudo-kin of the institution) to bypass traditional obstacles to free trade in land [Stone 1982].

The activity of the latter institution testifies to the tendency to evade the constraints of communal traditions; it is also, on the other hand, one of many proofs that land—the all-decisive productive asset—was not exchanged on real market conditions (cf. [Jakobson 1971]). Individualism and monetary relations dominated the economy, but capitalism was far away. None the less, the new economic structure caused multiple changes in the socio-cultural sphere.

Firstly, of course, business did not give up accounting and archives just because it was private. On the contrary, these spread to new social circles. Private letter-writing emerged, describing both private business and personal affairs—until then, only official letter-writing had been known. *Seals*, hitherto insignia of officials, became tokens of private identity. And of course, accountants and surveyors in private employment and street scribes writing down the personal letters for pay appeared, as did free-lance priests performing private religious rites.

Secondly, *individualism* itself took shape as a world view, manifesting itself not only in the private seal and the personal letter but also in the religious sphere and in art. While Ur III had consummated the transformation of the ordinary member of the primitive community into a subject of the state[31], the Old Babylonian era made him reappear as a *private man*.

On the other hand, Old Babylonian society was still a royal state. The king was, as during many preceding centuries, the largest estate owner, and directed many affairs while local autonomies when existing were restricted. A new duality had thus evolved, where clearly the »modern« aspect of society was the more vulnerable. Corresponding to the traditional royal aspect of society the ancestral royal ideology also survived, and in fact got its most famous expression precisely in this time: the preface and postface to Hammurapi's »law-code« (translated in [Pritchard (ed.)

[31] This is in fact part of the complaint of the minor rebellious gods in the *Story of Atraḫasīs* (above, note 27). While they were originally the »sons«, i.e. the lower-ranking members of the clan community, and the »chamberlain« thus nothing but the »elder« member governing common affairs, he has now become the master and they the dumb subjects.

1950:164-180]), where the king appears as sort of Bronze Age social democrat, assuring for his country affluence *and* justice. (The details of the text and the king's personality as it can be seen from his letters makes this look somewhat more honest than in Šulgi's comparable text).

The institution which connects this to the development of mathematics is the scribe school[32]. Before discussing the school itself, however, a brief remark should be made about language. Sumerian had been retreating as a spoken language already during Ur III, and maybe centuries before, as can be seen from the increasing dominance of Akkadian names. Official writing, it is true, persisted in Sumerian. In early Old Babylonian times, Sumerian was in all probability a dead language, and all non-scribal business was done in the Babylonian dialect of Akkadian. Official writing, always produced by one scribe and meant to be read by another scribe, was still made with some recourse to Sumerian: at times full and more or (often!) less grammatically correct Sumerian, at times staple Sumerian word signs used as abbreviations within otherwise Akkadian sentences. The Sumerian literary tradition, moreover, was transmitted in the scribal school, though increasingly in bilingual versions.

As to the school itself, its situation reflected that of the general economy. Some schools have been found within palace precincts, and may hence be regarded as official institutions. Others, however, have been located in living areas for scribes; they can hardly have been anything but private enterprise ([Lucas 1979:311f] offers a survey). In both cases, however, the students were trained for similar, »notarial«, accounting and »engineering« functions, i.e., for key positions in general social practice in private or official business[33]. Evidently, the *sine qua non* for any scribe was to master

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[32] Two fairly recent presentations are [Sjöberg 1976] and [Lucas 1979]. Older important general discussions are [Falkenstein 1953], [van Dijk 1953:21-27], [Gadd 1956], [Landsberger 1960], and [Kraus 1973:18-45]. Didactical texts illustrating various aspects of the school enterprise have been published and translated by Kramer [1949] and Sjöberg [1972, 1973, 1975].

[33] Mostly in public administration. »Scribes were limited to positions connected with administration or with substantial accumulations of private capital. Perhaps, also, they filled out contracts and legal documents at the gate of the city. If I were to make an intuitive sweeping estimate, I would say that perhaps seventy percent

the practical skills needed to perform these tasks.

Besides these skills, however, future scribes were taught to be proud of their profession. A number of texts have survived which were used in the school to inculcate professional pride. They tell us about the curriculum, but they also tell us which part of the curriculum was central for professional pride. The picture gained from these texts stands in significant contrast to actual scribal functions.

Firstly, indeed, the continuation of the Sumerian tradition beyond Hammurapi's time is, as formulated by Kraus [1973:28], »das größte Rätsel, welches der altmesopotamische Schreiber uns aufgegeben hat«. Scribes had to learn Sumerian because other scribes used Sumerian! Even more paradoxical, scribal school students were expected to speak the dead language with good pronunciation. Tradition alone will not do (though even the survival of traditions requires a motivation on the part of their carriers and hence an explanation), since the scribal school tradition appears to take a fresh start in the early Old Babylonian period (all the texts formulating its ideology belong to the second millennium).

Sumerian simply, however, is not the culmination of the scribal art. According to the »Examination Text A«[34], the accomplished scribe must know everything about bilingual texts; he must know occult writings and occult meanings of signs in Akkadian as well as Sumerian; he must be familiar with the concepts of musical practice, and he must understand the distorted idiom of a variety of crafts and trades. Into the bargain then comes mathematics, to which we shall return. All that, as a totality, has

of the scribes had administrative positions, twenty percent were privately employed, and the remainder became specialists in the diagnosis of illness, charms, magic, and other activities calling for some knowledge of writing«, as formulated by Landsberger [1960:119] in answer to a question whether the important role played by secret idioms of various crafts in the »Examination Text A« (see below) could correspond to future employment.

Employment outside the »notarial«, accounting and »engineering« sphere was clearly secondary: "A disgraced scribe becomes a man of spells«, we are told by a proverb [Lucas 1979:325].

[34] Ed., transl. [Sjöberg 1975]; cf. [Landsberger 1960:99-101]. Admittedly, the earliest extant copies of the text are quite late (*viz* Neo-Assyrian); as observed by Sjöberg, however, the contents of the text seem to require an Old Babylonian origin.

a name (of course Sumerian): nam-lú-ulù, »humanity« ([van Dijk 1953:23-26]; [Sjöberg 1973:125]).

True enough, the phenomenon has some similarity both to the practice of legalese and to the worst aspects of Modern humanism as a self-aggrandizing device for bureaucrats and court servants. Instead of making analogies, however, we may try to formulate an explanation starting from a more precise analysis of the Babylonian concept itself. We may then notice that *everything has to do with scribal practice*, but scribal practice transposed from the region of practical necessity into that of virtuosity. What appears from other didactical texts is that the scribe is expected to be proud, not of accomplishing his actual tasks but *of his identity and ability as a scribe*.

This connects scribal ideals to both aspects of contemporary general ideology. On one hand, the scribal function *as a whole* was by tradition a public function. If the King was to guarantee affluence and justice, who but the scribe was to do the job? On the other, the scribe was also *an individual*, a *private man*. In order to assure oneself of being something special, a *human being par excellence*, it was of course excellent to stand out as the one who gives the king prudent advice, and this is in fact part of scribal boasting [Landsberger 1960:98]. But there was not much satisfaction in pointing to trite everyday scribal activities, i.e., to the actual ways to »guarantee affluence and justice«. After all, phonetic Akkadian could be written with some 80 cuneiform signs. Everybody would be able to learn that. But everybody would not attain the level of virtuosity. Scribal professional *pride* needed something *really difficult* as its foundation; but the difficulties had to *belong at least formally to the territory of scribal tasks* if it was to serve *professional* pride. This, according to all evidence, is the reason for the specific configuration of Old Babylonian scribal »humanism«, and for its appearance as *art pour l'art*.

Another characteristic of the »examination texts« and related didactical texts should be mentioned before we leave the subject. In contrast to the picture presented by the Fara school texts they always appear to reflect a rather suppressive ambience—ever-recurrent in an early text (known as »Schooldays«) where the school-boy tells his experience of the day are the words »caned me« [Kramer 1949:205]. In »Examination text A« the student stands back as an ignorant dumbfounded by the teacher. Admittedly, it *is* the teacher who speaks through the text. But the double-bind situation which it suggests is still psychologically informative. The message seems to be that the scribe should be proud of being a scribe, but only privately; on service he should be a humble functionary knowing his place. Scribes were to be servants, not rulers and in reality rarely advisors of those in power. The scribe was to keep balance between actual loyalty and personal autonomy. His situation may have been similar to that of a Medieval clerk. Yet Renaissance humanism was as far ahead as capitalism; the Old Babylonian scribe was, after all, closer to the Fara scribe testing for the first time the possibilities of his professional tools than he was to Benvenuto Cellini.

VI. Mathematics

»The state« as a concept turned out to be subject to more dispute than presupposed by Wittfogel, my initial punching ball. What about mathematics?

Nowadays, of course, we know the meaning of the term inside our own world—at least until we are asked about borderline cases like accounting, engineering computation, magic squares or generative grammar. Well within the border we have a cluster of indubitably mathematical practices, disciplines and techniques, cohering through shared use or investigation of abstract, more or less generalized number or space or of other abstract structures.

Many single elements of this cluster can be traced far back in time, and be found in non-literate contexts, often at quite advanced levels. Currently, the term »ethnomathematics« is used about these elements when found in non-literate cultures [M. Ascher & R. Ascher 1986]. It is important to notice, however, that »ethnomathematics«, no less than »mathematics«, is *our* concept. The inhabitants of Malekula in Vanuatu would hardly have recognized the bunch of elements of their culture classified by us as »mathematical« as one entity. Their »kinship group theory« belongs more closely together with the kinship and marriage customs in general than with the drawing of closed patterns, which on the other hand belongs with the relation and passage between life and death[35]. Counting and the geometry of house-building will belong to still other domains.

Non-literate populations visited by modern ethnographers are not identical with the ancestors of Ancient civilizations; but it is a fair assumption that the mathematical techniques and practices of the latter constituted something similar in structure (or rather, lack of *own* structure) to ethnomathematics. Similarities may well have gone much further—as we shall see, graphs similar to those of Malekula were familiar in the Ancient Near East. If we are going to look for *mathematics* as *one* entity we may thus choose between two options: either we define one specific domain (traditionally number and counting) as being *their* mathematics, which will allow us to postulate the existence of mathematics far back into an indefinite past; or we may decide (as I intend to do) that the distinctive characteristic of mathematics as *one* entity is the *coordination* of several abstracting practices.

The choice of coordination as the defining feature does not free us from all arbitrariness. It is still a question, e.g., whether counting and addition are one or two practices; if they are two, the introduction of addition is already mathematics, since it cannot be done in isolation from counting. So, I shall end up by defining the transition to mathematics as the point where *preexistent and previously independent* mathematical practices are coordinated through a minimum of at least intuitively grasped understand-

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[35] The abstract marriage algebra of Malekula is described by M. Ascher & R. Ascher [1986:137-139], the graph-theoretically refined closed patterns by M. Ascher [1988:207-225]. The disconnectedness between the two does not imply, of course, that the intellectual training gained through graphs cannot have made it easier for the informant to formulate the principles of marriage rules explicitly for the benefit of the ethnographer.

Ascher & Ascher [1986:132] make the point that the »category mathematics is our own« but stop short of drawing the same conclusion about ethnomathematics, for fear perhaps of devaluating the non-literate cultures which they discuss. This caution should be superfluous: the elements of ethnomathematical thought are no more random or isolated than our elements of mathematical thought—their connections are different.

ing of formal relations. Remaining ambiguities I shall accept as an unavoidable ingredient of human existence.

VII. From tokens to mathematics

The earliest mathematical technique which can be attested in the Near East is represented by small objects of burnt clay found as far back as the late ninth millennium B.C. and still present in the proto-literate period[36]. From early times, a variety of shapes are found: spheres, rods, cones, circular disks, more rarely other shapes. Many types are found in two sizes, and in certain cases the objects are marked by various incisions. During the fourth millennium, the number of shapes and of extra varieties created through multiple incision proliferates violently.

Because of continuity with later metrological notations (on which below), the objects must be tokens, i.e., *tangible symbols* for other objects—normally goods of economic importance, it appears. Obviously, the tokens constitute a *system* of symbols, used all over Iran, Iraq, Palestine and Turkey.

The emergence of the system appears to coincide with the change to agricultural subsistence [Schmandt-Besserat 1986:254]. Agriculture itself, of course, will have had no need for symbolization, nor will barter of grain for obsidian (or whatever exchange can be imagined). The most plausible suggestion for the function of the token-system is supplied by the excavation of a fifth millennium site (Tell Abada) in east-central Iraq [Jasim & Oates 1986:352]. Tokens are found in several places; yet groups of varied tokens (e.g., 8 spheres, 4 cones, 1 disc, one rod) contained together in

[36] Denise Schmandt-Besserat, who discovered the widespread appearance and high age of a system which until then had only been recognized in the later fourth millennium, has published a long array of papers on the subject, of which I shall only refer to the original publication [1977], an early popularization [1978], and a recent paper [1986] discussing *inter alia* social and cognitive interpretations. Another recent publication on the matter to be mentioned is Jasim & Oates [1986].

vessels are found only in one place, but there repeatedly: in the most important building of the village, which according to a number of infant burials may have had religious functions, but whose many rooms shows it not to be a mere shrine (or »temple«). Most likely, it was also a communal storehouse, the heart of a religiously sanctified redistributive system which was moving toward taxation in favour of responsible personnel, and within which the tokens have served for accounting [Schmandt-Besserat 1986:268f].

This interpretation is supported by other evidence. Tell Abada is not the only place where the tokens turn up in non-residential buildings [*ibid.*, 254]. Moreover, tokens (or, rather, prestige versions of tokens made in stone) are also found as high-status grave goods from the sixth millennium onwards,, e.g. in the fourth millennium site Tepe Gawra (near Ninive)—in the grave apparently possessing the highest status 6 stone spheres constitute the total deposit [Schmandt-Besserat 1986:255]. Admittedly, Jasim and Oates [1986:351f] mention this as an argument for non-accounting functions of the objects; more plausible, however, is Schmandt-Besserat's explanation ([1986:269] and, in more detail, [1988:7f]) that the occurrence of tokens in the deposits of high-status burials reflects a high-status position for those who administered by means of tokens while living; their presence in infant graves in Tepe Gawra and elsewhere, furthermore, suggests that the manipulation of tokens was (or belonged with) a hereditary function (as burial deposits in children's graves are normally taken by archaeologists as evidence for hereditary social ranking)[37].

Due to later continuity the meaning of certain tokens can be interpreted. So, a disk marked with a cross appears to stand for a sheep (and two disks for two sheep). Most, however, are uninterpreted or only tentatively interpreted, while the principles involved are only subject to limited doubt. They can be illustrated by Schmandt-Besserat's suggestion that a small cone

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[37] It may be objected that we would not expect so highly developed stratification in the beginning of the Neolithic. Some indications exist, however, that the ecology of the Near East was rich enough to support stratified settlements and to call for organized redistribution as early as the late Mesolithic Natufian, and that ranking and even hereditability of high status had developed by then (see [G. A. Wright 1978:218-221]).

stands for a specific measure of (i.e., a specific type of basket or jar containing) grain, a small sphere for another, larger measure / container, and large cones and spheres for still larger measures [1986:268]. Other types might signify other staple products (dried fruit, oil, wool, ...). We observe that the marked disk stands for both quality (sheep) and quantity (one) at the same time; the same holds for the cone if representing the *grain*-contents of *a specific container*. There is no symbol for *abstract number* or for *volume as such*. Since the containers for grain and for oil were different, »volume concepts« had to be specific. Measure only exists as »natural measure«, and number only as »concrete number«[38].

The fourth millennium proliferation of the number of token types corresponds to the need of the more highly organized economy of social systems like that of the Susiana plain. New commodities had to be handled, and those of old to be followed in more detail (from later evidence we may guess, for instance, that »sheep« would be differentiated into ewes, rams and male and female lambs). In addition, the tokens were now used as »delivery notes« for goods sent from the periphery to Susa, enclosed in sealed containers made of clay (»bullae«)[39].

A disadvantage of the sealed bulla as a bill of lading was that it had to be broken in order to be »read«. A solution, however, was at hand: before the tokens were put into the bulla they were pressed into its surface, each leaving a clearly visible impression. The observation that thereby the

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[38] Evidently, this cannot be read out from the tokens themselves. It follows from an agreement between general ethnomathematical experience and the reflection of the token system in proto-literate metrologies.

One question which cannot be solved in this way is whether »bundling« was included into the system. *If*, e.g., a small disk corresponded to an animal, would then a large disk correspond, e.g., to »a hand« (5) or »hands and feet« (20) of animals? Would a »sphere-container« be supposed to contain a fixed number of »cone-containers«? At some point in the development such bundling was introduced, but we have no means to assure that it had already happened in the Neolithic.

[39] At this point we begin to approach hard facts. This last-mentioned use of the tokens follows from the geographical distribution between Susa and lower-ranking settlements of seals, broken sealings, bullae prepared for use but not yet closed, and dispatched bullae (see [Wright & Johnson 1975:271]).

enclosed tokens had become superfluous will have called forth another step: the replacement of the hollow bulla by a flattened lump of clay where the impressions could be made (by tokens or, rather, by styli able to make similar impressions) and over which the cylinder seal could be rolled. These are the first genuine *clay tablets*, normally known as »numerical tablets«; like the bullae, they are found in Susa and the Susian orbit as well as in Habuba Kabira, the Uruk V-outpost (those of Uruk are found in rubbish heaps and cannot be dated)[40].

As carriers of information, the numerical tablets had an important advantage over the bullae: their surface could be structured, first by distinguishing the four edges of an approximately square tablet and next by dividing the surface into compartments through incised lines. Another advantage was discovered in Uruk IV: through pictographs *quality* could be separated from, or added to, *quantity*. A drawn circle with a cross was used to indicate sheepness, and impressions looking like pictures of small and large cones and spheres were used to indicate the number of sheep[41].

The whole development from the introduction of bullae with impressions of tokens and seals to the creation of the pictographic script was evidently coupled to the development of a complex society and to the needs of statal administration for more precise controls, as it was delineated above. It was no consequence of state formation *per se:* as pointed out already, the control involved in state formation need not be bureaucratic control. But the development was a consequence of *state formation as it actually happened in the Sumero-Susian area,* and we may assume that it was

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[40] See [Le Brun & Vallat 1978:47, 57] for Susa and [Jasim & Oates 1986:349] for Habuba Kabira.

[41] Readable expositions of the various facets of the development are given by Nissen [1985] and by Damerow, Nissen & Englund [1988, 1988a].

It should be observed that the sequence bulla—numerical tablet—pictographic tablet is in the main derived from the inner »logic« of the process combined with indirect arguments rather than from direct stratigraphic criteria: because only numerical and no pictographic tablets are found in Habuba Kabira, this setlement must be earlier than Uruk IV, where pictographic writing *is* attested. But then, since bullae and numerical tablets are found in Habuba Kabira, they must be earlier than pictographic writing; etc.

the age-old connection between sanctified unequal redistribution and token accounting which made bureaucratic control a natural corollary of the further change of the redistributive system toward taxation.

Improvement of book-keeping is an improvement of a mathematical technique, which was thus an effect of state formation. But book-keeping alone *does not constitute mathematics*.

On the other hand, mathematics *did* emerge in the process, and even in the form of multiple coordination. Firstly we may look at the metrological sequences and number systems used in the texts. These were first analyzed thoroughly by Jöran Friberg [1978], whose preliminary results have now (on the whole) been confirmed and expanded through computer analysis as part of the Berlin Uruk project [Damerow & Englund 1987].

The first thing to be observed about these systems is that counting is still concrete. In fact, although the basic signs (varied through combination in various ways and addition of strokes) are pictures of the small and large spheres and cones[42], a number of different systems are in use, with different relations between the visually identical signs.

Firstly, there are *two sequences for counting*[43]. One (the »sexagesimal system«) starts by a small cone (»1«), continues by a small circle (»10«), a large cone (»60«), a large cone with an impressed small circle (»600«), a large circle (»3600«), and culminates with a large circle with an impressed small circle (»36 000«). This system, characterized by its systematic shift

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[42] In principle, the appearance of the signs could be an accidental result of the fact that these are the impressions which can be made by vertical and inclined impression of a thin and a thick circular stylus; the existence of bullae where the tokens actually contained are impressed [Schmandt-Besserat 1986:256] suggests, however, that the similarity between tokens and signs is not accidental, and that the circular stylus was chosen precisely because it could so easily produce the desired impressions.

[43] A sequence »for counting« is characterized by a separation of quantity from quality, as, e.g., in our »3 sheep« or »6 m«. A »metrological sequence«, on the other hand, has quality inherent in quantity (as in »mmmm« instead of »4 m«).

Throughout the history of Mesopotamian mathematics this distinction remains less clear than the historian of mathematics might prefer. Instead of our »4 m«, e.g., an Old Babylonian scribe would usually have written »4«, expecting everybody to know that lengths are measured in this unit.

between the factors 10 and 6, is used to count slaves, cattle, tools made from wood or stone, vessels (standing for a specific measure of their customary content), and probably lengths.

The other main counting system (the »bisexagesimal system«, with units in the ratios 1:10:60:120:1200:7200, i.e., successive factors 10, 6, 2, 10, 6) is used to count products related to grain (rations? bread?), and certain other products.

Besides, *three metrological sequences* have been identified. One is used for capacity measures for grain. If the basic unit is *B* (a small cone), the next are 6 *B* (small circle), 60 *B* (large circle), 180 *B* (large cone) and 1800 *B* (large cone with inscribed small circle)—the factor sequence is thus 6, 10, 3, 10. We observe that both order and ratios differ from those of the sexagesimal number system.

Another metrological sequence (testified only in Uruk III / Jemdet Nasr) is used for areas. It was still in use in far later times, which allows us to interpret the small cone as an iku (c. 60m·60m). Then follows a small cone with inscribed small circle (6 iku), a small circle (18 iku), a large circle with inscribed small circle (180 iku) and a large circle (720 iku) (factor sequence 6, 3, 10, 6).

A third metrological sequence is of unidentified use.

Obviously, all sequences are based on the principle of bundling, which demonstrates that principles derived from counting were applied to the regularization of natural measures. Apart from that (admittedly important) step, however, the plurality of sequences and the absence of any system in the succession of the same symbols and in the sequence of ratios is hardly a proof that the career of mathematics had begun.

This beginning, however, is demonstrated by closer investigation of features not yet mentioned. Firstly, what I have just described is just one part of the sequences, from the »basic unit« upwards. This is the part whose signs derive from the old token system, and which may therefore be of indefinitely older age—even though it is not implausible that the counting notations and the area notation were fresh creations, taking over the symbols of the grain system and adapting them to the actual bundling steps of the verbal counting systems and to the area metrology in use (on areas, see below). The other part consists of fractional sub-units, which are positively new. In the counting sequences, the first sub-unit (»$^1/_2$«, in the sexagesimal system, and in specific contexts perhaps »$^1/_{10}$«) is symbolized by the small cone turned 90° clockwise, which would evidently make no sense for freely rattling tokens. In the grain system, a first step is made in a similar fashion[44], producing »$^1/_5$ *B*« (=»*C*«). In a second step downward, »$^1/_n$ *C*« (n=2, 3, 4, probably 5 and possibly higher values) is symbolized by n small cones arranged in a rosette. (No area units below the iku are attested, but this may well be because such smaller units do not occur in allocations of land—our only epigraphic evidence for area metrology). This involves an knowledgeable application of »inverse« counting to metrological innovation, and must thus be characterized as *mathematics* as defined above.

Another metrological innovation based on mathematical premeditation pertains to the calendar—more precisely, one of the calendars[45]. Until much later, indeed, the »time-keeping calendar« is a luni-solar calendar, whose months are on the average 29$^1/_2$ days, shifting between 29 and 30. Of these months there are 12 to a year, and about every three years an intercalary month is inserted in order to adjust the year to the tropical and agricultural year. To the meticulous Ur III administration, months of changing length were unacceptable, as we may easily imagine, and a system was employed where the overseer was responsible for pressing 30 days worth of work out of each worker per month, irrespective of its real length, and got food and fodder rations for his workers and animals according to the same principle. Now, through fastidious analysis of certain proto-literate herding texts Robert Englund has been able, firstly, to confirm an interpretation of the time-keeping notation proposed by Vaiman [1974] on intuitive grounds, and secondly to show that the Ur III administrative calendar was in reality a proto-literate invention and practice.

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[44] The sign itself, it is true, differs from the turned picture of the cone used in the counting sequence: it might look as a picture of the half- or quarter-sphere tokens, and could thus have been present already in the token-system. But like the fractional counting number, it is turned 90° clockwise, indicating that both are conceptualized as belonging to the same (»fractional«) category.

[45] The following description of Sumerian and proto-literate timekeeping is built on Robert Englund's pioneering work on administrative timekeeping [1988].

The notation combines the pictogram showing a sun half raised above the horizon with strokes (counting the years), ordinary sexagesimal numbers (months) and sexagesimal numbers turned 90° clockwise (days). Already for the reason that these distinctions only make sense when the symbols are fixed in clay will this be a fresh invention of the proto-literate period. The free creative manipulation of several sexagesimal counting systems demonstrates mental independence of context-bound counting and ability as well as resolution to combine different elements of mathematical thought in order to create an adequate tool[46].

Similarly, even the creation of a counterfactual calendar in order to attain mathematical regularity can be seen legitimately as evidence of coordination, *viz* between bureaucratic organization and mathematical thought. It will also involve at least an intuitively based decision that the rounding error was not larger that acceptable. On both accounts the administrative calendar thus testifies to the emergence of genuine mathematics.

All this had to do with the complex of counting, metrology and accounting. A final observation involves geometrical practice in the network.

We have as yet no direct proof that the area of a rectangular field was calculated from its length and width—none of the texts which appear to indicate lengths and widths contain area information. But two pieces of indirect evidence can be found. Firstly, the same area system (or at least an area system with the same factor sequence) is known from later times to be strongly geared to the length unit[47]. Thus, the basic area unit is the sar, which is the square of the fundamental unit of length (the nindan or »rod«, equal to c. 6 m), but whose name (presumably meaning a »garden

[46] A similar albeit weaker observation could be made from the existence of »dependent metrological sequences« produced from those described above through addition of strokes and used to count or measure specific varieties of the goods counted or measured by the corresponding fundamental system—for instance, to measure emmer instead of barley. In this case the innovation may go back to the late pre-literate creation of supplementary token types (and token sequences?) by means of incisions.

[47] See Powell [1972], the principal reference for Sumerian area measures.

plot« [Powell 1972:189-193]) suggests an independent origin as a »natural unit«. The iku itself is a square ešé (the ešé, meaning a »rope«, being equal to 10 nindan). Further on in the sequence, the bur (=18 iku), again appears to have originated as a »natural unit«.

This suggests that the system emerged from a process of mathematical normalization, where natural seed or irrigation measures were redefined in terms of length units, thus stabilizing the system, as Powell points out [*ibid.*, 177]—and since the upper end of the sequence is already present in Uruk III (where no area units below the iku are testified but may still have existed), the redefinition must have taken place already then.

The other piece of indirect evidence is a proto-literate tablet referred to by Damerow & Englund [1987:155 n.73]. It deals with a surface of which the two (identical) lengths and the two (slightly different) widths are told. Calculating the area by the »agrimensor formula«[48] one finds a nice round value: 10 times the highest area unit, i.e., c. 40 km^2. The implausibly large value tells us that we have to do with a school exercise, and the improbability to hit upon the round value by accident suggests that the exercise was constructed so as to achieve it, and thus that the area had to be calculated as done in later times.

Area measurement is not the only element of geometrical practice attested in the proto-literate period. Already the ground-plan of the late pre-literate »Limestone Temple« [E. Heinrich 1982:74 and Abb. 114], perhaps even two fifth millennium temples [*ibid.*, 32 and Abb.71, 74], possess a regularity which suggests architectural construction. Remains of a ground-plan left under an early Uruk IV (or possibly late Uruk V) temple, moreover, shows that it was carefully laid out by coloured string ([Heinrich 1938:22], cf. [Heinrich 1982:63, 66]). One of the many different groups of experts present in proto-literate Uruk must hence have been architects skilled in practical geometrical construction[49]—and since only

[48] I. e., average length times average width. This method was used in the computation of the area of not too irregular quadrangles at least from ED III to Old Babylonian times, and even far into the Middle Ages.

[49] Like the idea of writing (but not the script itself), this technique also seems to have been borrowed by the proto-Elamite culture (which had a centre in Susa but had others far into the Iranian East, and which was more or less contemporary

»official« prestige buildings suggest the existence of a geometrical plan, they must have worked exclusively for the Temple.

We can also be reasonably sure that the planning of buildings and of building enterprises will have involved computation of brickwork and man-power requirements. Firstly, a culture which defines a specific administrative month for the sake of fodder calculations would hardly take the enormous costs of prestige building just as they came. Secondly, the evidence for precise geometrical lay-out coupled to the standard brick demonstrates that calculations could be made, as indeed they were in later times; it is plausible that this was even the idea behind the mutual adjustment of standards. If so, however, the computation of areas and volumes from linear dimensions will have arisen already in the architects sphere, and the gearing of area measurement to measures of length will also have involved the architectural branch of practical geometry. Proto-literate mathematics will already have coordinated number and metrical space—one and the other, we may safely assume, as practical concerns and not as abstract fields of interest[50].

The formation of mathematics as a relatively coherent complex was thus concomitant with the unfolding of the specific Uruk state. Is that to say that it was a direct consequence of statal bureaucratic rationality—sort of modified and attuned Wittfogel thesis, mechanistic-functionalist though on revised premisses? Hardly. Other early bureaucratic states have existed

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with Uruk III). This follows from Beale's and Carter's careful analysis [1983] of the geometry of the proto-Elamite architectural complex of Tepe Yahya IVC, in which base-lines separated by integer multiples of a standard measure (equal to 1.5 times the standard brick length) define the exterior edge of outer walls and the mid-lines of inner walls. Apart from a different choice of ratio between the standard measure and the standard brick, moreover, the same code appears, e.g., in buildings from Habuba Kabira (the Uruk V outpost mentioned in chapters III and VII).

[50] One field which was not yet integrated (and which never was until the modern era) was »ethnomathematical graph theory«, cf. [M. Ascher 1988]. That it was none the less present we may infer from somewhat later evidence: in the Fara tablets such »graphs«, complex symmetric patterns drawn by a continuous line, turn up time and again—see the specimens in [Deimel 1923:31] (broken)); [Jestin 1937: CLXXX, #973]; and [Edzard 1980:547].

without producing similar results[51], and bureaucratic management of agriculture would probably have been better served by natural measures (as suggested by the changes in Babylonian metrology after the mid-second millennium). Bureaucracy itself does not demand the type of coherence inherent in the Uruk formation of mathematics. What is involved is, we might say with Weber, a *particular* spirit of bureaucracy, one tempted by intellectual and not by *merely bureaucratic* order. We also find it expressed in the *lexical lists*, which are *more than a means of teaching the script:* they also provide an ordered cosmos, and a cosmos of a specific sort: putting wooden objects together in one category, vessels in another, etc., amounts to what Luria [1976:48ff] labels »categorical classification«, in contradistinction to his »situational thinking«[52]. Still, the lists *are* a means for teaching, and thus a vehicle not only for literacy but also for the »modern«, abstracting mode of thought—precisely the mode of thought preferring mathematical coherence to situationally adequate seed measures, etc. The latter part of their message will have supported, and have been supported by, the development of the main administrative tool: the clay tablet with

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[51] A beautiful example seems to be presented by the linear B tablets of the Mycenaean palace bureaucracy. Even though Mycenaean art bears witness of a strong and inquisitive interest in geometrical regularity [Høyrup 1983] there is to my knowledge no evidence whatsoever of a transformation of scribal accounting arithmetic into *mathematics*.

[52] Illustrated, e.g., by this dialogue [*ibid.*, 55]:

Luria, explaining a psychological test: »Look, here you have three adults and one child. Now clearly the child doesn't belong in this group".

Rakmat, an illiterate peasant from Central Asia: »Oh, but the boy must stay with the others! All three of them are working, you see, and if they have to keep running out to fetch things, they'll never get the job done, but the boy can do the running for them [...]«.

Situational thinking was found in Luria's investigation of prevailing modes of cognition in Soviet Central Asia to be »*the* controlling factor among uneducated, illiterate subjects«, while both modes were applied (with situational thinking dominating) among »subjects whose activities were still confined primarily to practical work but who had taken some courses or attended school for a short time«. »Young kolkhoz activists with only a year or two of schooling«, on the other hand, employed the principle of categorical classification »as their chief method of grouping objects«.

its ordered formats[53].

In so far as the emergence of mathematics is to be ascribed to a particular Uruk variant of the bureaucratic spirit, this spirit was thus interacting intimately with, and largely a consequence of, the school organization of teaching (whose typical features we already encountered in a mathematical exercise). If a complex process is to be reduced to a simplistic formula, the emergence of mathematics was called forth neither by technical needs nor by the bureaucratic organization or by writing *per se*, but *only through the interaction of these with each other and with that school institution which provided recruits and technical skills to the bureaucracy*.

VIII. Trends in third millennium mathematics

As long as the Sumerian city-states remained dual societies, mathematics was on the same side as writing and bureaucracy. Throughout the third millennium, therefore, the career of mathematics runs parallel to that of expanding bureaucratic systems, spreading literate activities, and improved writing. In so far as all this was a simple continuation of the trends inherent in the proto-literate state, mathematics too was a continuation.

Let us first look at metrology. It may wonder that no metrological sequence for weights has been mentioned above (unless, of course, the unidentified sequence contains weight units)—especially in view of the fact that metal smelting is actually attested in Uruk [Nissen 1974:8-11]. But technical activities of this sort were not the concern of accounting, and whatever the craftsmen have done was not committed to writing and thus subjected to mathematical regularization[54].

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[53] This problem of the interplay between tool and mode of thought I shall not pursue any further in the present connection, only refer to its position as the central theme in [Damerow & Lefèvre (eds) 1981].

[54] Import of metals will of course have been a matter of bureaucratic interest. But nothing so far known suggests that archaeologists have come upon tablets from

Later, when copper and silver acquired monetary functions, on the other hand, weight became an accounting concern *par excellence*. In the beginning of ED III, thus, the weight system *is* well attested. A consequence of this late development of weight metrology is a high degree of mathematical systematization (see [Powell 1971:208-211]) in the shape of »sexagesimalization«, adoption of the fixed factor 60 from the principal (in ED III the *only*) counting system, in analogy with what had already happened in the proto-literate creation of the calendar notation. Starting from the top, a »load« (some 30 kg, the Greek »talent«) is divided into 60 mana, each again subdivided into 60 gin (the later šekel). The gin is subdivided into še, »barleycorns«, which in real life weigh much too little to fit another sexagesimal step; but 180=3·60 še to a gin agrees fairly well with real barley.

Sexagesimalization was not the preserve of the weight system. In general, when pre-existent systems were extended, it was done »the sexagesimal way«. So, e.g, 60^3 and 60^4 were added to the counting sequence; the gin was transferred from weights to other systems in the generalized sense of $^1/_{60}$; and established systems were expanded upwards through multiplication of the largest traditional unit by sexagesimal counting numbers. This development is most straightforwardly explained as the natural consequence of the situation that mathematics was *already present* as a coherent way of thought, both *in actu* and as impetus and challenge, carried by continuing school teaching.

Another perceptible trend is parallel to that of centralized reforms of writing and bureaucratic procedures (and, though only on the ideological level, to the recurrent idea of a »social reform«): intentional and methodical changes of metrology in order to facilitate bureaucratic procedures. This is of course analogous to the proto-literate introduction of the administrative calendar; the instance which is best certified in the pre-Ur III period is the Sargonic introduction of a new capacity measure in the order of the barrel, the »gur of Akkad« of 30 ban = 300 sila (≈ 300 l) instead of the current gur of 24 ban = 240 sila and the Lagaš gur of 144 sila (see [Powell 1976:423], where the advantages of the new unit in connection with

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the archive of trade.

computations of rations are discussed).

A third trend, finally, is akin to the appearance of literary texts, and like literary text it begins in the Fara period, concomitantly with the emergence of the scribal profession as a separate group. We might speak of a first instance of *pure mathematics*, namely, of mathematical activity performed in order to *probe the possibilities* of existing concepts and techniques and neither for immediate use in practice nor for plain training of skills to be used in practice.

The evidence is constituted by the oldest mathematical exercises after those of the proto-literate period, which could only be distinguished from real-world accounting and mensuration by the occurrence of round and implausibly but not impossibly large numbers and by the lack of the name of an official carrying responsibility for the transaction [Friberg 1990:539]. One of the Fara problems ([Jestin 1937, #188]; unpublished analysis by Jöran Friberg) is *almost* of the same type, with the difference that now the area involved is rather *impossibly large*. Two other Fara texts [*ibid.*, #50 and #671] require that the content of a silo containing 2400 »great gur«, each of 480 sila, be distributed in rations of 7 sila per man (the correct result is found in #50: 164 571 men, and a remainder of 3 sila; the solution of the other tablet is wrong or at best uncompleted—analysis of the two texts and of the method used in [Høyrup 1982]). A fourth text (analyzed by Jöran Friberg [1986:16-22]), comes from the Syrian city Ebla (whose mathematics was avowedly taken over from the Sumerians) and is presumably of slightly later date. It deals with the successive division of 100, 1000, 10 000, 100 000 and 260 000 by 33 (concretely: if 33 persons get 1 gubar of barley, how much barley do you count out for 100, 1000, 10 000, 100 000 and 260 000 persons?).

Apart from being division problems and from the »impossibly large« numbers of rations dealt with, the three last problems have one decisive thing in common: the divisors are *irregular*, they fit the metrologies and number systems used *as badly as possible* (Ebla spoke a Semitic language and had decimal number words, but combined these in writing with the Sumerian sexagesimal system; 33, of course, is irregular on both accounts). As Jöran Friberg [1986:22] puts it,

(I) the fact that three of the four oldest known mathematical problem texts[55] were concerned with exactly the same kind of »non-trivial« division problems must be significant: the obvious implication is that the »current fashion« among mathematicians about four and a half millennia ago was to study non-trivial division problems involving large (decimal or sexagesimal) numbers and »non-regular« divisors such as 7 and 33.

A number of school exercises dating between the Fara period and Ur III (mostly Sargonic) have been identified (see [Powell 1976]). Some of them are characterized by the occurrence of »impossibly large« numbers, e.g., a field long enough to stretch from the Gulf to central Anatolia. There is no trace, however, of continued interest in »pure mathematics«—which, in view of the striking statistics cited by Friberg, must be significant. As literary creativity, once a scribal exploration of the possibilities of a professional tool, was expropriated by the royal court as a political device, so also mathematical exploration appears to have vanished from a school more directly submitted to its bureaucratic function in a society loosing its traditional dual character. Two verifiable forces survived as determinants for the development of »school-and-bureaucracy mathematics«: sexagesimalization and systematization governed by the dynamics of internal coherence; and regularization determined by the requirements of bureaucratic efficiency.

A small and isolated tablet found on the floor of a Sargonic temple suggests that a third force *may* possibly have operated outside the school-and-bureaucracy system. More on this below in connection with the Old Babylonian development, to which it is connected (note 69).

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[55] Apparently for rhetorical reasons, Friberg discards the proto-literate school exercises which he himself has been the first to identify.

IX. The paramount accomplishment of bureaucracy

Waning duality dwindled further in Ur III, the school-and-bureaucracy complex reached a high point, and so did bureaucratic and accounting rationality. No wonder, then, that Ur III brought about the culmination of the tendencies of late ED and Sargonic mathematics.

We already encountered Šulgi's administrative reform above, and we remember that metrological reform was presented as a cornerstone in his establishment of »justice«. Another, mathematically more decisive part of the administrative revolution was the development of the conceptual and technical tools for the many calculations inherent in the reform.

First of all a new number notation was created as a final outcome the process of sexagesimalization: the sexagesimal *place value system*, which permitted indefinite continuation of numbers into the regions of large and small. The idea had been in the air for several centuries, as demonstrated firstly by the generalized use of the gin in the sense of $^1/_{60}$, and next also by the particular idiom of a late Sargonic school exercise discussed by Powell [1976:427], where a »small gin« is introduced for $^1/_{60}$ of $^1/_{60}$. But precisely the use of *names* for the fractional powers shows that the system was not positional, and was not extendable *ad libitum*. We can thus be fairly sure that the introduction of place value does not antedate Ur III[56].

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[56] This conclusion is not changed by the claims and the partially new text material presented by Whiting [1984], who conflates place value notation with what I have here called »sexagesimalization«. But Whiting's evidence underscores how much was in the air in the actual computation techniques in use at least since the Sargonic era, and his explanation of two apparent writing errors in a pre-Sargonic tablet of squares (OIP 14,70, transliterated and translated in [Edzard 1969]) suggests that an idea similar to the gin-tur was used already in the 25th c. B.C.

The errors so abundantly present in the computations on which Whiting bases his argument, on the other hand, make it obvious that the system after which calculators were groping was *not yet at hand* as more than an inherent possibility—similarly, perhaps, to the way the decimal place value system may have been potentially present in the Chinese use of counting rods for perhaps 2000 years before

The Mesopotamian place value notation was a pure floating-point-system, with no indication of absolute place (in the likeness of a slide rule); it could thus only be used for intermediate calculations—in accounting, one sixth of a workday, e.g., would be designated »10 gin« [Powell 1976:421] in order to avoid misunderstandings. For this reason, only very few indubitably Ur III tablets carry indubitable place value numbers[57], though some do (one instance is discussed [*ibid.*, 420]).

The important point about the place value notation is not the possibilities it offers in additive and subtractive accounting, where the disadvantage of a double number system will have outweighed the ease of writing which it brought about. It lies in the multiplicative domain, in the possibilities of the system to surmount the conflict between mathematical and technical rationality (as discussed in connection with the tendency of proto-literate scribes to prefer mathematical coherence to practical orientation), and to do this more radically than could be done by changes in the metrological system. If a platform had to be built to a certain height and covered by bricks and bitumen, e.g., changes in length measures could not be made which at the same time would facilitate manpower calculations for the earth- and brickwork, the computation of the number of bricks to be used, and the consumption of bitumen. But once the place value system was available, tables could do the trick. A »metrological table« could be used to transform the different units of length into sexagesimal multiples of the nindan. A table of »constant factors« would tell the amount of earth carried by a worker in a day, the number of bricks to an area unit, and the volume of bitumen needed per area unit. With these values at hand everything was a question of sexagesimal multiplications and divisions, which again

giving rise to the genesis of a genuine place value notation (see [Martzloff 1988:170f, 181-184]), and to the way it was demonstrably mimicked by the Greek idea of *pythmens* (see Pappos, *Collectio* II.1, in [Hultsch 1876:I,2]).

[57] In the integer range between 1 and 599, place value and »normal« administrative notation cannot be distinguished. Therefore, the scribe did not need to decide whether he used one or the other in such cases, nor can we settle the question.

A few undated tables of reciprocals (see below) probably belong to Ur III, but the paleographic distinction between Ur III and Old Babylonian tablets is not very safe for tablets containing exclusively or predominantly numbers.

were facilitated by recourse to tables, this time tables of multiplication and of reciprocal values. The conflict between »natural« and »mathematical measure« was solved similarly in other domains, and so well solved that supplementary technical measures could be introduced *ad libitum*, as indicated by an apparent proliferation of brick systems. This was the great advantage of the system in a society where the scribes were financially responsible overseers of all sorts of productive activities.

It is a fair guess that the place value system was probably invented with the purpose to solve these problems, but since we do not possess the memoirs of the inventor we cannot know[58]. What can be known is that other highly adequate place value systems are known historically to have spread at a snail's pace, in processes taking hundreds of years or even longer. If the invention was not made in Šulgi's think-tank (something like the administrative department of Kraus' conjectured *Hofkanzlei*), a central decision must at least have been made to propagate the system through the scribe school, which must thus have been under centralized control (as one would guess anyhow, given the character of Ur III society and Šulgi's interest in having the school teach what his scribes needed).

Much the same could be said about other aspects of the administrative system, especially about the introduction of a system of *balanced accounts*, at times with automatic cross-checking[59]. The school provided the administration with accountants and calculators whose collective competence has hardly been equalled by any comparable body before the 18th or 19th century (CE, for once!). Judged on the purely utilitarian premisses inherent in the Šulgi hymns cited above, the Ur III school did everything that could be done.

It is remarkable, then, that no trace whatsoever is left of non-utilitarian

[58] So much was in the air, indeed, that the most difficult step was not to get the idea in itself but to find the courage to do so. For an isolated inventor (be he practical calculator or teacher) the system would be worthless. Only when backed by tables of constants, reciprocals etc., and thus only when large-scale use made it economically feasible to produce these, were place value numbers any good.

[59] See [Høyrup 1980:19f, and 85f notes 39, 42 and 44], which contains cross-cultural comparison, whose references for Ur III book-keeping itself, however, are partly outdated. The most recent treatment of the subject is given by Englund [1990:13-55].

mathematical interests from the period. Not only are texts lacking, which in itself proves nothing, since no school texts at all from the period have been identified. More decisive: an investigation of the mathematical terminology of the subsequent Old Babylonian period shows that terms used for current operations of utilitarian calculation are Sumerian; the key terms of the non-utilitarian branches, on the other hand, are Akkadian, and the oldest non-utilitarian texts formulate even the additive and subtractive operations (for which current Sumerian terms existed of course) in Akkadian—with the exception of the finding of reciprocals and the extraction of square-roots, which referred to tables in the Ur III tradition, and the traditional Sumerian terms for which were even adopted as loanwords and provided with Akkadian declination[60]. According to all evidence, Ur III thus managed to bring its scribes to a high level of mathematical competence without engendering any sort of pure-mathematical interest, i.e., without leading to any intellectually motivated investigation of the possibilities of professional tools beyond the needs of current business—in contrast to the situation in Fara, where much more modest competence did call forth »pure« investigation. Borrowing an expression from a classical discussion of other aspects of the Mesopotamian intellect [von Soden 1936], Ur III demonstrates »Leistung und Grenze« of the early bureaucratic state as a promotor of mathematical development.

X. The Culmination of Babylonian mathematics

The vast majority of Mesopotamian genuine mathematical texts come from the Old Babylonian period. Before Marvin Powell and Jöran Friberg began their work, almost nothing was known from the third and fourth millennia, and no system whatsoever had been noticed in the meagre material (even the connection between the Ur III administration and the

[60] The details of the argument build on my investigation of Old Babylonian »algebra« [Høyrup 1990].

creation of the sexagesimal system was only suggested as a conjecture by Powell in 1976). From the 1300 years separating the Old Babylonian from the Seleucid period, again practically nothing was known (since then, Jöran Friberg has located a few items). Finally, a small number of texts with Seleucid dating had been published. No wonder that the Old Babylonian period was considered the culmination of Babylonian mathematics, which in histories of mathematics was simply identified with this climax.

In part, this is certainly a consequence of the source situation. As there is some though not full continuity from Old Babylonian to Seleucid mathematics, something *must* have existed in the intermediate years. Yet today, when at least a sketchy picture of the state of the mathematical art in the early and the intermediate period can be made, Old Babylonian mathematics is enforcing its particular character upon us in more real terms: never before, and never after, was comparable depth and sophistication achieved in Ancient Mesopotamian mathematics. Even the source situation seems to reflect realities and not merely the random luck of excavators and illegal diggers: after the Old Babylonian period the institutional focus for the production of sophisticated mathematics disappeared.

Why is that? What *was* the make-up of Old Babylonian mathematics? And what was its purpose?

First of all, Old Babylonian, quite as much as third millennium mathematics, spells *computation*. All texts compute something, they *never prove* in Euclidean manner, and they *only explain* through *didactical discussion of specific examples* of computation.

Many computations are purely utilitarian, and for good reasons. The texts are scribe school texts (the teacher's copies, not students' solutions as most of the pre-Ur III texts which have come down to us); and graduate scribes, as we remember, would normally go into notarial jobs, where they needed little but accounting mathematics, or into engineering-like occupations, where a wider range of practical geometry etc. would be required[61].

[61] It seems likely that some specialization was present. According to Landsberger ([1960:97]; cf. [1956:125f]), indeed, the Old Babylonian »lexical lists distinguish, according to degree of erudition and specialization, fifteen varieties of *dubsar* or scribe« which, however, all disappear in the subsequent period, together with the scribe school. The evidence is insufficient, however, to decide to which extent the

Utilitarian mathematics was thus a continuation of Ur III mathematics, involving sexagesimal calculation, the use of the tables of metrological conversion and of »constant factors«, knowledge of accounting and surveying procedures and of computational techniques at the level of the rule of three, familiarity with the computation of areas and (occasionally fairly intricate) volumes[62]. All this, in fact, is found, often in complex combinations as in »real scribal life« where the manpower needed to dig a trench and carry off the dirt was more interesting than its volume.

Just as important in school, however, were non-utilitarian computations, to judge from the statistics of extant texts. Dominating in this field was a domain traditionally denoted »algebra« by historians of mathematics, and which is in fact homomorphic with second- and higher-degree equation algebra of the Medieval and Modern epoch. The designation can be argued to be problematic, both because a literal reading of the terms of the Old Babylonian discipline indicates that it does not deal with number but with areas (quite literally: with fields), and because a close investigation demonstrates that the methods used were indeed sort of »naive« (i.e., reasoned but not explicitly demonstrative) cut-and-paste geometry[63].

Many problems belonging to this category look fairly abstract. For instance, we may be given the sum of the length and the width (l+w) of a rectangular field and the sum of the area and the excess of the length over the width (A+(l-w)), and then be asked to compute the length and the width (AO 8862, in MKT I, 108f, cf. interpretation in [Høyrup 1990:

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job specialization was reflected in specialized school curricula.

It should be observed that dub-sar NIG.ŠID, translated »mathematician« by Landsberger [1956:125], should rather be understood as »accountant«.

[62] Often of course by means of what we would call »approximate formulae«, forgetting in this distinction that even the most exact area formula becomes approximate when the terrain surveyed is hilly and no Euclidean plane.

Karen Rhea Nemet-Nejat (forthcoming, chapter III) presents a survey of practical problem types occurring in the Old Babylonian mathematical texts.

[63] [Høyrup 1990] presents the arguments for this interpretation in philological and mathematical detail, while [Høyrup 1989] presents an overview. [Høyrup 1985] is a fairly complete but preliminary and rather unreadable exposition (»It is difficult to follow the red thread—provided there is any«, as Asger Aaboe put the matter).

309ff]). In this case, only the remark that »I went around it« tells that the person stating the problem speaks of a real field; other problems are even more deprived of the smell of real life. Still others, however, attach themselves directly, e.g., to military engineering practice, as may be illustrated by this example[64]:

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[64] BM 85194, rev. II.7-21—ed. MKT I, 149. The translation is mine, and builds on my reinterpretation of the Old Babylonian mathematical terminology (I have left out indications of restituted damaged passages and corrected a few copyist's errors tacitly). Without going into irrelevant details, the following explanations should in principle make the text comprehensible for those who want to wrestle with a real piece of fairly complex Babylonian mathematics:

1) Numbers are written in a sexagesimal place-value system (Neugebauer's notation).
2) Horizontal extensions (length, breadth) are measured in the unit nindan (≈6 m).
3) Vertical extensions are measured in kùš (cubits), where 1 kùš = $^1/_{12}$ nindan ≈ 50 cm.
4) Volumes are measured correspondingly, in the unit sar = nindan^2·kùš, here left implicit (»gán« is not the unit but an indicator of category and loose order of magnitude).
5) To »append« designates a concrete addition, and to »tear out« the corresponding concrete subtraction.
6) To »detach the igi of *n*« means finding its reciprocal ($^1/_n$)—actually looking it up in the table of reciprocals.
7) »To raise« means calculating a concrete entity through multiplication, as done, e.g., in operations involving proportionality.
8) To »double« designates a concrete process—in the actual case the doubling by which a rectangle is produced from a right triangle.
9) To »break« denotes a bisection into »natural« or »customary« halves—as, in the actual case, one side of a triangle is customarily bisected when its area is calculated.
10) To »make *a* surround« means constructing a square with side *a*; if we do not care about the real (geometric) method of the Babylonians we may translate it »to square«.
11) The »equilateral« of an area is the side which it produces when laid out as a square; in numerical interpretation, its square root.

Some hints can be found in MKT I, 186. Those who want to apply the geometrical interpretation (not given in MKT) should be aware that a rectangle *n* [cubit high] by *n* [nindan long] is dealt with as a square; i.e., the units which anyhow are left implicit are disregarded. Cf. Høyrup 1985: 56.

7. Of dirt, 1,30,0 (sar), gán. A city inimical to Marduk I shall seize.
8. 6 (nindan) the (breadth of the) fundament of the dirt. 8 (nindan) should still be made firm before the city-wall is attained
9. 36 (kùš) the peak (so far attained) of the dirt. How great a length
10. must I stamp in order to seize the city? And the length behind
11. the *ḫurḫurum* (the vertical back front reached so far?) is what? You, detach the igi of 6, the fundament of the dirt—0;10 you see. Raise 0;10 to
12. 1,30,0, the dirt—15,0 you see. Detach the igi of 8—0;7,30 you see.
13. Raise 0;7,30 to 15,0—1,52;30 you see. Double 1,52;30—
14. 3,45 you see. Raise 3,45 to 36—2,15,0 you see. 1,52;30
15. make surround—3,30,56;15 you see. 2,15,0 from 3,30,56;15
16. tear out—1,15,56;15. What is the equilateral? 1,7;30 you see.
17. 1,7;30 from 1,52;30 tear out—45 you see, the elevation of the city-wall.
18. $\frac{1}{2}$ of 45 break—22;30 you see. Detach the igi of 22;30—0;2,40.
19. Raise 15,0 to 0;2,40—40, the length. Turn back, see 1,30,0, the dirt. Raise 22;30,
20. $\frac{1}{2}$ of the elevation, to 40, the length—15,0 you see. Raise 15,0 to 6—
21. 1,30,0 you see, 1,30,0 is the dirt. The method.

To a first inspection, this looks like a slightly idealized piece of engineering mathematics: a siege ramp formed like a right triangular prism is to be constructed, and we know certain parameters concerning the structure and have to find the others. (The minor blunder that an already given value is asked for again, instead of another which is actually found, will be due to an editor-copyist's mixup with other problems dealing with the same configuration—one follows on the same tablet).

A second look, however, changes everything. The construction has already started; we already know how much dirt is going to be used for the ramp, as well as the height already reached and the remaining distance. But we do not know the intended total length or final height of the ramp, nor the length of the part built so far! The outcome, after intricate geometrical considerations, is a problem of the second degree.

Evidently, such a problem would never present itself to a surveyor in real life. In fact, no single second-degree (or higher) problem in the texts solves a problem which could be encountered in practice, nor can any be imagined within the Babylonian horizon. And yet, such problems were extremely popular (the same unfinished ramp, for example, turns up in another tablet making use of a somewhat different terminology and thus probably produced in a different school)[65]. Definitely, mathematics needed not be applicable in order to acquire high status within the curriculum—*if only it looked applied*, as the above puzzle from the engineers' wonderland.

This may look as a paradox. Why should evidently »pure« mathematics be disguised as applied? Neugebauer [1954:790], obviously disgusted, speaks of »educational artificiality which fancies it is making simple geometrical problems more appealing by using practical examples containing unreal examples«. Why should pure mathematics be restricted to computation? And why on earth should a school for future clerks, managers and engineers make so much of the training of useless skills?

The answers have to do with the position of the scribal profession and the role of the scribal school. Like the writing of phonetic Akkadian, accounting mathematics and trite computations of prismatic volumes were too uncomplicated to serve as foundation for professional pride. In order to demonstrate *virtuosity*, Akkadian had to be supplemented by Sumerian and secret writing, and the volume computation had to be turned around into a second-degree puzzle. Higher »algebra« was thus the expression of scribal »humanism« corresponding to the numerate aspect of the scribal vocation (and a choice expression), as Sumerian was the expression corresponding to the literate aspect. The important thing about second-degree »algebra« was not that it could not be used; the distinctive characteristic was that is was *complex*, i.e., *non-trivial*. The situation repeats that of the Fara scribes on a higher level, whose investigation of the possibilities of writing produced the first literary texts, and whose comparable experiments with their computational tools produced »pure« division problems.

But virtuosity had to be *scribal* virtuosity in order to serve *professional* pride (which would of course be the only sort of pride at which a scribal school could aim). Therefore, even complex mathematical problems should

[65] More precisely: such problems were popular according to their place in the corpus of texts and thus in the curriculum. There is no particular reason to believe that average students liked them. To the contrary, the generally suppressive character of the examination texts might suggest that mathematics was, within scribal humanism no less than in 19th century (CE) German neohumanism, also accepted because of its disciplining effects.

belong at least in form to the category of scribal problems. Though »pure« *in substance*, scribal mathematics was by necessity *applied in form*.

Strictly speaking, furthermore, the numerate aspect of the scribal venture was not *mathematical* in a general sense but *computational*. The virtuoso scribe had to be a virtuoso in *finding the correct number*. *Pure mathematics* in the sense which we have derived from the Greeks was not open as an option. Only *pure computation* would make the day[66].

Finally, the scribe was a *practitioner*, no philosopher or teacher. In Babylonia as everywhere else, the main thing for a practitioner is to be able to handle his methods aptly and correctly. In mathematics at the Old Babylonian level, this requires more than a *modicum* of understanding[67]. But in all vocational training then as now, apt and correct handling of methods is learned primarily through *systematic training supported by explanation*, not vice versa—as it was once formulated, you do not extinguish a fire by lecturing on the nature of water. Though transmission of *methods* was the central *aim* of the school, the *solution of* (adequately selected) *problems* was thus necessarily the *central teaching mode*—as, again, real, practical problem solution was the ultimate purpose of the training of *utilitarian* methods.

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[66] The situation was certainly going to be different in the Middle Ages, even for professional groups ressembling the Old Babylonian scribal profession. By then Greek mathematics was *already at hand*, and »scribal« computation could (and would, in the Islamic and Christian worlds) be seen as a special instance of that lofty enterprise. What is at stake here is the option of *inventing* something like Greek mathematics, which was a task quite different from that of assimilating the *Elements*—cf. the analysis of the former process in [Høyrup 1985a:17-30].

[67] Truly, quite a few historians of mathematics have supported the view that it was based on a tool-kit of recipes found empirically and assimilated by the scribes through rote learning—a view mostly based on familiarity with one or two problems quoted in translation in some semi-popular exposition. Scholars really familiar with the sources have always known that Babylonian mathematics could only have been produced by people who understood what they were doing, and they have supposed that oral explanations will have accompanied the terse expositions written in the tablets. During my own investigation of the sources I have located a couple of texts which in fact contain this fuller explanation (see [Høyrup 1989:22-25], and [Høyrup 1990:299-305, 320-328]).

Nothing thus remains of the supposed paradox when it is seen in the light of Old Babylonian scribal humanism. But another important characteristic persists which should be discussed. The »unfinished ramp« illustrated the »humanist« character beautifully, through exaggerating features which are present yet less conspicuous in other problems. But exaggeration is, already by definition, untypical, and so also in this case. The text is so much of a riddle that we can almost hear the real wording of lines 9-10 as »[...] *Tell me, if you are a clever scribe,* how great a length must I stamp in order to seize the city?«. Most texts, however, are much more terse and, more important, the majority of those which contain several problems are fairly or even highly systematic (with the exception of some late Old Babylonian anthology texts—the »unfinished ramp« is known from precisely these). The riddle shows the family likeness between Old Babylonian »pure computation« and »recreational mathematics«, which before it became a column in newspapers and mathematics teachers' journals was a »pure«, virtuoso outgrowth of practitioners' »oral mathematics«[68]. But systematization is of course foreign to any genre of campfire riddles, mathematical as well as non-mathematical. The systematics of the Babylonian mathematical texts reflect that school system in which they were composed, and that tendency to establish »bureaucratic order« even in the intellectual realm which had characterized it since the proto-literate period[69]. If many of the distinctive »humanistic« characteristics of Old

[68] The relation between practitioners' mathematics and recreational problems is discussed in [Høyrup 1987:288-290], and again more fully in [Høyrup 1990a], which also takes up the »scholasticized« character of Old Babylonian »pure« mathematics.

[69] Evidently, this difference in kind between recreational and scribe school mathematics does not preclude that a scribe school in need of non-trivial problems and corresponding methods borrowed them from a non-literate, recreational tradition. Evidence exists that this is precisely what happened:

Firstly, it is characteristic that the key terminology of the early »algebra« texts is Akkadian (as is in principle the whole Old Babylonian mathematical corpus even in texts where Sumerographic shorthand and Sumerian technical terms abound). In one text the quadratic completion, the essential trick in the solution of second-degree equations, even seems to be designated »the Akkadian« (*viz.*, Akkadian method; see [Høyrup 1990:326]). No doubt, thus, that »algebra« was no heritage from the Sumerian school tradition.

Babylonian mathematics must be explained with reference to the particularities of the carrying school institution as a relatively autonomous institution in an individualistic culture, its over-all character of *mathematics* was still guaranteed by the traditional character of the school as developed in interplay with the bureaucratic state.

The dependence of Old Babylonian mathematics on the school-and-bureaucracy-complex and its characteristic double-bind conditioning, on the other hand, was also the factor which effectively inhibited the emergence of *theoretical* mathematics of the Greek kind. As I have formulated it elsewhere [1990:337], the scribal school was »only moderately inquisitive and definitely not critical«. This befitted the education of future »humble officials knowing their place« yet proud of their social status. In later times

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Secondly, at least a cognate of second-degree »algebra« predates Ur III. Another favourite problem, indeed, shares part of the characteristic terminology (and, presumably, the naive-geometric technique) with the »algebra«: the bisection of a trapezium by a parallel transversal. The oldest known specimen of this problem, however, is the tablet mentioned in the end of chapter VIII, which was found on the floor of a Sargonic temple (see [Friberg 1990:541]).

The problem is so specific that independent reinvention is unlikely. But if the school has not transmitted the problem and its solution, who has? My best guess is an Akkadian surveyor's environment, which can quite well have existed in central Mesopotamia in the early Old Babylonian epoch, to the north of that Sumerian core area where graduates from the scribe school may possibly have had a monopoly of surveying.

Interestingly, one Sargonic school exercise (A 5446, see [Whiting 1984:65f]) seems to presuppose knowledge of a basic »algebraic« identity. It asks for the areas of two squares of side R-r, where R is a very large, round measure, and r a very small unit. Without knowledge of the identity $(R-r)^2=R^2-2Rr+r^2$ (which of course follows easily from geometrical considerations), the calculation will be extremely cumbersome.

Even in the Sumerian South, it should be added, scribal monopoly on surveying and geometrical practice is not too firmly established. Krecher [1973:173-176] points out that Fara contracts for purchases of houses involve a »master who has applied the measuring cord to the house« (um-mi(-a) lú-é-éš-gar), while a »scribe of fields« (dub-sar-gána) is involved when land is bought; a Sargonic document groups together the »surveyor« (LU_2.EŠ.GID), the »scribe« (dub-sar) and the »chief of the land register« ($\text{SA}_{12}.\text{DU}_5$). Krecher supposes even the »master« and the »surveyor« to be scribes, but in particular concerning the latter we cannot know for sure.

similar institutions would provide fairly suitable vehicles for the transmission of the ultimate outcome of the Greek mathematical endeavour; yet the dawn of the Greek endeavour itself was too dependent on non-suppressive critical discussion to be within the reach of a scribal school culture.

XI. Devolution

I shall finish my discussion of Mesopotamia by some cursory and undocumented remarks on the state, the scribal profession and the development of mathematics after the end of the Old Babylonian period, before passing to some even more brief comparative observations.

The end of the Old Babylonian epoch inaugurated the dissolution of much of the complex which, according to the above, had shaped and even engendered Mesopotamian mathematics.

Firstly of that sort of state which, since its emergence as a pseudo-redistributional organization, had guarded the pretense to be the upholder of justice and affluence (in spite of often contrary realities). The end of the Old Babylonian epoch was brought about by the Hittites, who sacked Babylon, after which a warrior people (the Kassites) took over power in the Babylonian area. They exploited it, not by taxation however vaguely disguised as redistribution but by direct extorsion, as conquerors would mostly do until the advent of the more sophisticated methods of the Modern era, taking over part of the land, allying themselves with the autochthonous upper class and pressing tribute from a re-communalized peasant class. City life, on that occasion, did not disappear *completely*—but the proportion of town to country dwellers reverted to the level of the pre-literate Middle Uruk period.

The scribe school disappeared. Administration and scribes were still needed, but scribes were from now on trained as apprentices inside their »scribal family«.

The self-asserting individualism of the Old Babylonian period disappeared. The particular scribal expression of Old Babylonian individualism, »humanism«, disappeared, too. Instead, scribal pride was founded on the membership in an age-old *tradition*. That cloak of magic and secrecy which Wittfogel ascribes to the bureaucrats of the managerial and functional state is in fact a product of the intellectual crisis caused by its breakdown.

Even mathematics disappeared—at least from the archaeological horizon. But realities are involved, too. Techniques, of course, survived. But the few texts of later times suggest that the integrity of mathematics as a subject on its own disappeared—while Old Babylonian mathematical texts would contain *nothing but* mathematics, things were now mixed up. »Pure« scribal interest in mathematics disappeared, it seems. The evidence suggests, indeed, that second-degree algebra, even though it turns up again in a few Seleucid texts, survived in a practitioners' (surveyors' and/or architects') rather than in the scribal environment[70]. Moreover, the evolution of metrology suggests that technical mathematical skills declined. As explained above, the routines and procedures associated with the place value system overcame the conflict between mathematical and technical rationality, thus making the use of »natural measures« unnecessary. From the Kassite era onwards, however, the metrological system changes; field measures keyed to the squared length unit, e.g., are replaced by seed measures. Apparently, technical efficiency was no longer compatible with »mathematical efficiency«, i.e., coherence and simplicity.

In the Late Babylonian epoch (from c. 600 B.C. onwards), finally, mathematics reappears above the horizon. Its practitioners are no longer primarily scribes, i.e., accountants and engineering managers; instead, they designate themselves as »exorcists« (*āšipu*) or »priests« (*šangû*). The latter title, oddly enough, coincides with the Sumerian sanga, who was not only a priest but also a manager of temple estates and a teacher in the Fara

[70] For one thing, the set of Sumerian equivalents for Akkadian technical terms changed—tab, once used as a Sumerogram for *eṣēpum* (»to double« or »repeat concretely«—arithmetically, to multiply by an integer *n* below c. 10) came to designate addition. It thus appears that the scribes translated the language of »algebra« into their favourite Sumerian tongue for a second time without knowing that (or without knowing too precisely *how*) it had been done before.

school; the Late Babylonian *šangû*-mathematician, however, was no practical manager as his proto-literate predecessor but an astrologer.

The astrologer-priests who created Late Babylonian mathematical astronomy performed technical wonders, no doubt. Their skill in developing interpolation schemes, six-place (sexagesimal places!) reciprocal tables etc. is impressing. But if we understand mathematics as »a coherent way of thought, both *in actu* and as impetus and challenge«, then the high point of Mesopotamian mathematics was reached in Hammurapi's Bronze Age and never again.

XII. Supplementary comparative observations

A possible test of the plausibility of the theses advanced above on the connections between the specific process of state formation and development and the emergence and shaping of mathematics would be cross-cultural comparison. Implicitly, of course, much of the analysis *is* already cross-culturally based, through the use of theoretical tools sharpened on non-Mesopotamian whetstones. In the present appendix I shall only point to two possibilities of explicit comparison.

First, of course, Egypt suggests itself, as every time a mirror is to be held up to Mesopotamia[71]. State formation in Egypt was roughly contemporary with that of Uruk, presumably slightly later. Its background, however, was more explicitly in agreement with Carneiro's warfare model. Pharaoh united Egypt through conquest, and courtly art demonstrates that he was proud of that. The early Egyptian state was not built on any redistributive pretext or ideology[72].

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[71] Apart from the general literature, the following builds in particular on [Baines 1988]; [Brunner 1957]; and [Høyrup 1990b].

[72] Or, at least, only on redistribution in an utterly distorted form (cf. [Endesfelder 1988]): Pharaoh took hold of the societal surplus and redistributed part of it to his officials while returning perhaps promises of cosmic stability to the general peasant

Writing was also roughly contemporary, and presumably slightly later. But until late in the Old Kingdom, literacy was extremely restricted, and not before the Middle Kingdom, i.e., in the outgoing third millennium, did a scribal school arise.

If no other forces were present which could nourish the process, we should thus expect the development of mathematics as a coherent whole and, especially, of »pure« orientations, to be much slower than in Mesopotamia. As far as it can be judged from the meager evidence from the Old Kingdom, this seems indeed to be the case. Firstly, some generosity is already required to see the development of the Egyptian unit fraction system as evidence of a »pure« orientation; but even if that is granted one will have to observe that the unit fraction system seems only to be created *as a system* in the Middle Kingdom, in the wake of the new scribe school institution. No other branch of Egyptian mathematics can at all be considered non-utilitarian. Secondly, it is questionable how far the unification of single techniques into a coherent whole had developed before it was definitely brought about by the unit fraction system.

While Ancient Egypt is a mirror through which the Mesopotamian development is recurrently observed, Medieval Western Europe is rarely mentioned as an analogue. In one important aspect, however, the Medieval West *is* relevant, *viz* as a dual society. If Gilgameš shares essential features with the Homeric kings, he can also be compared to a Frankish warrior-king. The Church, on the other hand, shares with the Sumerian Temple the status of a purported institutionalization of the common good; to a large extent, its incomes derived from benevolent gifts (often compulsory, it is true, and in the case where the gift was a nobleman's donation of land with appurtenant peasants it could only be made productive through continued compulsion; but these details are irrelevant for my present purpose, and nobody knows whether realities were much different in Sumer). The interesting thing is that *literacy was until the High Middle Ages the exclusive ally of the ecclesiastical »Temple Institution«;* except for a few dreamers of learning like Charlemagne and Otto III one can describe the history of Central Medieval learning without ever presenting the feudal

population.

power in person. »Who was the feudal lord who donated land for the Cluny monastery? It doesn't matter: feudal lords did that sort of thing!« »Who pacified the French core areas in the late 10th century to the extent that cathedral schools could revive? No single lord or king, it was part of a general trend visible in many places«; etc. And vice versa of course: in the *Poema de Mio Cid*, the tale of this most Christian hero of the Spanish *reconquista*, the role of the Church is as secondary as in *Gilgameš and Agga*. Societal duality is thus a recurrent historical type in state formations not yet fully satisfying Runciman's criteria (quotation F), and literacy and learning belong with the institutionalization of alleged general interest, not with the warrior-robber lordship in its interaction with a pre-state, communal or kinship-based sector.

The Medieval parallel can be pursued further, into the High Middle Ages. Then, as we know, duality was reabsorbed, and royal centralization was well served by literate clerks. But at the same time the environment of learning, rapidly growing and therefore less directly subject to the »Temple« institution, went through a process of intellectual emancipation, first in the »twelfth century renaissance« and then in the universities. But scholars remained *clerks*, firstly because of the general socio-cultural and the particular institutional context, secondly because of the future social position of most university students. As in the Old Babylonian scribal school, though less strictly, the traditional binding to the »Temple« and the actual nexus to scribal (notarial and cameralistic) functions in existing society set limits to the tendencies toward intellectual enfranchisement.

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