Torben Braüner, **Hybrid Logic and its Proof-Theory**, Applied Logic Series Volume 37, Springer, 2011, pp. XIII+231. ISBN: 978-94-007-0001-7 (hardcover) EURO 99,95, ISBN: 978-94-007-0002-4 (eBook) EURO 99,99.

Modern hybrid modal logic came into being in the 1990's, but derives directly from Prior's work on temporal logic in the 1960's. Possible world semantics is formulated in terms of states and accessibility relations, which are essentially graph-theoretic notions, but modal languages are based on modal operators—necessity, possibility—that do not refer to states. If modal constructs are understood temporally, then to a first approximation semantics talks of moments of time, while formulas talk about after and before. Yet we commonly bring moments of time into our language: "At 3:00pm the President gives a speech." Hybrid modal logics provide direct reference to possible worlds in the formal language, similar to the temporal "3:00pm" just mentioned. This increases expressiveness by incorporating what had been purely semantic notions into the proof theory. The machinery for doing this involves *nominals*, propositional letters that are true at exactly one possible world. Then a nominal can be thought of as a 'name' for the unique world in which it is true. So for starters, hybrid logics add nominals, and interpret them as world names.

Nominals are not enough by themselves—a way is needed for them to interact with other formulas. At a minimum one also adds a *satisfaction operator* to the language: if a is a nominal and φ is a formula, $@_a\varphi$ is read, "at state a, φ is true," and is interpreted semantically accordingly. Here is an example of a formula in this language, $@_a\varphi \supset @_b@_a\varphi$. A little thought establishes the validity of this formula.

Nominals plus satisfaction operators allow semantic machinery to be internalized quite generally. For example, if a and b are both nominals, $@_ab$ asserts that b is true at state a, and since nominals must be true at unique states, this really says that a and b designate the same state state. Similarly $@_a\Diamond b$ tells us that state b is accessible from state a. In short, we have equality and accessibility expressible within the language.

Other, more powerful, machinery is also considered, giving a hierarchy of logics of increasing strength and expressibility. A very natural addition is $\downarrow$, where $\downarrow x\,\varphi$ is taken to be true at a state w if φ is true at w with x assigned the value w. In other words, $\downarrow$ binds its nominal to the point of evaluation. (Technically, x must be a nominal *variable*, and valuation functions are needed in the semantics, but the intuitions are clear). One can think of $\downarrow$ as storing information and @ as retrieving it. Quantification over nominals—possible worlds—can also be considered. The full range is discussed in the present book.

The example of the three o'clock Presidential speech, above, says that allowing nominals is often quite natural. It also turns out to be of considerable technical advantage. Frame conditions such as irreflexivity, that otherwise could not be captured axiomatically, can be captured easily using nominals and satisfaction ($\neg @_a a$). Tableau systems become more uniform, instead of the usual jumble of *ad hoc* devices. Many modal logics lacking interpolation theorems now have them. It seems like much gain, and through intuitively natural machinery. Nonetheless, hybrid logics are still not as well-known in the modal community as they should be. The book

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Modal Logic, Blackburn, de Rijke, and Venema, Cambridge, 2001 contains a highly recommended presentation. The book under review here continues, and specializes, this. It assumes the general ideas of modal logic are known, and concentrates on aspects of proof theory peculiar to hybrid machinery. It presents in a uniform manner results of the author and collaborators that have appeared in papers published from 2004 to 2008, in addition to material from the author's *Stanford Encyclopedia of Philosophy* article on hybrid logic, from the *Handbook of Modal Logic* article by the author and Silvio Ghilardi, and from the author's thesis for the Danish higher doctorate. It should be noted that the disparate background is not apparent—the present book is a coherent, unified, and very readable entity.

A unique feature of the book is its focus on natural deduction systems and normal form theorems. Elsewhere in the literature hybrid axiom systems are common, as are tableau/Gentzen systems. In the present book each system of hybrid logic being considered is given a natural deduction formulation, and a constructive proof of normalization is presented. Axiom, tableau, and Gentzen systems are also given, but their completeness is derivative from that of a basic natural deduction system. This fully justifies the inclusion of the phrase "Proof Theory" in the title of the book. Throughout the discussion is clear, informative, and natural. It can be recommended as a book to read, as well as to consult, after a basic exposure to hybrid logics.

The book begins with an introductory chapter, explaining the ideas and the history of hybrid logic, including formalism and fundamental results. It then turns to a detailed examination of propositional hybrid logic, and includes a useful discussion of the motivations behind natural deduction systems. Soundness and a detailed proof of completeness are given, along with a normalization result specifically tailored to hybrid logics. This is worth the price of admission itself. Then attention turns to Gentzen style systems and axiom systems. As noted earlier, this pattern continues throughout.

There is a chapter (3) devoted to propositional hybrid tableaus and decision procedures—with various combinations of hybrid machinery considered. These tableaus are somewhat analogous to prefixed tableaus or labeled tableaus (as in 'labeled deduction systems'). The important difference is that instead of involving extra machinery that is not part of the formal language, everything that occurs is a formula, and nothing more. There are no signs, prefixes, etc. For example, in a prefixed tableau system one sees $\sigma\,\varphi$ where φ is a formula, but σ is a *prefix*, a kind of possible world name whose syntactic structure encodes information about an accessibility relation, but is not part of the formal logical language. Similarly for labeled tableau systems. But in hybrid tableaus, what takes the place of such things is $@_a\varphi$. Of course $@_a$ references a possible world, but $@_a\varphi$ is a formula of the language itself, with no outside machinery. This is aesthetically and formally significant, and a good example of the advantages of hybrid machinery. Analytic cut issues are discussed. Many examples are given. The presentation is continued in Chapter 9, where a full comparison is made between the labeled approach and the hybrid approach.

A minor point of terminology. Theorem 3.5 is called a *Model Existence theorem*. The theorem is a tableau version of what is most commonly called a *Truth lemma*. The Model Existence Theorem is something else again—there are such things for some modal logics, but I don't know about their status when nominals come into it.

Chapter 6 and especially chapter 7 are of particular interest—they cover first-order hybrid logic. First-order modal logic itself is a complex thing. One can have *actualist* (varying domain) or *possibilist* (constant domain) quantifiers. One can have *intensional* constants and variables that can change their designation from world to world. One can have various combinations of such things. When intensional (non-rigid) terms are allowed, scope distinctions become especially important, and machinery has been introduced in the literature to deal with this. The author proposes a particularly simple mechanism. Non-rigid terms are, as usual, thought of as being modeled by functions on states—an idea going back to Carnap. Then the $@_a$ operator is extended to apply to non-rigid terms, so that $@_a i$ denotes what term i evaluates to at state (nominal) a. This machinery is related to 'lambda abstraction' which is another mechanism to handle non-rigidity, partly developed by the reviewer, and the book discusses this connection—formulas involving predicate abstraction can be reformulated entirely within first-order hybrid logic with $@$ and $\downarrow$. This give a different way of thinking—a different set of insights—into intensional modal logics. Extensions of the machinery are discussed which allow non-rigid terms to be partial—not designating at all states. The whole approach is a very natural one in a hybrid context.

Chapter 8 examines a hybrid version of intuitionistic modal logic. This is essentially a bimodal logic, with the intuitionistic part having a Kripke-style semantics that one can think of as epistemic, and a modal part also having a Kripke-style possible world semantics. Such things have been investigated before—quite a bit, in fact. The key new material here involves the hybrid approach. As usual, this is elegantly done, and as usual it is organized around natural deduction and normalization. Various modifications are examined, including one incorporating Nelson's paraconsistent logic.

The book ends with a somewhat philosophical discussion entitled "Why does the proof-theory of hybrid logic behave so well?". I will not try to summarize the author's points. I will say I enjoyed the discussion. And the book.

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